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ON PROPULSION, GUIDANCE AND CONTROL
IN EFFECTING SATELLITE RENDEZVOUS

WILLIAM JOHN BRANDEL, JR.
and
ROBERT LOUIS WILDEY

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ON PROPULSION, GUIDANCE AND CONTROL IN
EFFECTING SATELLITE RENDEZVOUS

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Submitted in Partial Fulfillment
of the Requirements for the
Degree of Master of Science
at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June, 1961

ON PROPULSION, GUIDANCE AND CONTROL IN
EFFECTING SATELLITE RENDEZVOUS

by

William John Brandel, Jr.

and

Robert Louis Wildey

Submitted to the Department of Aeronautics
and Astronautics on May 20, 1961, in partial fulfillment of the requirements for the degree of
Master of Science.

ABSTRACT

This investigation analyzed families of ascent trajectories to determine total velocity requirements, division of velocity impulses and the expected errors in position and velocity upon arrival in the terminal region caused by uncertainties in the position, magnitude and direction of the velocity vector at burnout of the booster stages.

Conditions at burnout of the booster and apogee of the trajectory were solved using an IBM 650 computer; the error analysis was accomplished by numerical integration on IBM 709 equipment repeated up to the apogee time of the zero-error trajectory.

From the tabulated and graphical results presented herein, some trade-offs between propulsion requirements and the accuracy of guidance and control systems can be seen.

Thesis Supervisor: Paul E. Sandorff

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ACKNOWLEDGEMENTS

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Little of the reference material could have been located if it were not for the efforts of Miss Katherine Livingston of the M.I.T. Libraries.

This work was done in part at the M.I.T. Computation Center.

TABLE OF CONTENTS

<u>Chapter No.</u>		<u>Page No.</u>
	Object	1
I	Introduction	2
II	Statement of the Problem	6
III	Computational Methods and Procedures	17
IV	Results	27
V	Conclusions	52

Figures

1	Definition of Elements Used in Computational Procedures	17
2	Platform Coordinates and De- finitions of Booster Burn- out Errors	26
3	Summation of Booster Burnout Velocity and Injection Velocity Increment.	28
4	Effect of Altitude and Range Angle on Booster Burnout Velocity and Injection Velocity Increment	30
5	Effect of Errors in the Magni- tude of the Booster Burnout Velocity on Range and Range Velocity Demonstrating Linearity	33
6	Effects of Range Error at Booster Burnout on Position at Zero-error Apogee Time	37
7	Effects of Range Error at Booster Burnout on Velocity at Zero-error Apogee Time	38

<u>Figures</u>		<u>Page No.</u>
8	Effects of Altitude Error at Booster Burnout on Position at Zero-error Apogee Time	39
9	Effects of Altitude Error at Booster Burnout on Velocity at Zero-error Apogee Time	40
10	Effects of Error in Magnitude of Booster Burnout Velocity on Position at Zero-error Apogee Time	41
11	Effects of Errors in Magnitude of Booster Burnout on Velocity at Zero-error Apogee Time	42
12	Effects of In-plane Misalign- ment of Booster Burnout Velocity on Position at Zero- error Apogee Time	43
13	Effects of In-plane Misalign- ment of Booster Burnout Velocity on Velocity at Zero- error Apogee Time	44
14	Effects of Track Error at Booster Burnout on Position and Velocity at Zero-error Apogee Time	45
15	Effects of Lateral Misalign- ment of Booster Burnout Velocity on Position and Velocity at Zero-error Apogee Time	46
16	Effect of Platform Definition on Lateral Position Error	50
References		54

OBJECT

It is the object of this thesis to present in graphical form the effects of errors in position and velocity at booster burnout on the position and velocity of a rendezvous vehicle at zero-error apogee time.

CHAPTER I

INTRODUCTION

Requirements

Long before the launching of the first earth satellite the military, scientific, economic and political ramifications of such a vehicle were considered and discussed. Each of the above considerations generates adequate reasons for a satellite rendezvous.

The most important and probably the most immediate requirement is in the field of the military. In orbit about the earth today are satellites capable of reconnaissance; little additional equipment is required to extend this capability to ferret and jamming missions or ICBM early warning. Interrogation and possible negation of such enemy vehicles are missions that require a satellite rendezvous.

Satellite stations in space require the physical exchange of personnel, equipment and fuel. Completion of such an exchange requires a "soft" rendezvous and continuation of the two vehicles in the same orbit until the transfer is complete.

Both civil and military organizations are developing communications satellites; these devices involve much complex and expensive equipment that will, in time, require repair or replacement. Satellite rendezvous is a solution to this problem.

Communications satellites are given special consideration in this report by the inclusion of data for orbit altitudes as great as 20,000 nautical miles above the surface of the earth. Earth satellites much above this altitude are of little interest since they are subject to large perturbations by the sun and moon and hold no advantage over the satellite with a period of 24 hours. The effect of such perturbations and the difficulties to be encountered in maintaining a high altitude orbit are discussed by Edelbaum in reference 1.

Background

The subject of satellite rendezvous has received its share of attention in the past several years. Literally hundreds of papers and articles have been written on the various phases of rendezvous and interception. Most of these works are very general and deal primarily with ideas rather

than figures and hardware. Other authors look into the problem in detail and find it to be enormous; it seems to take a well-funded project to generate any amount of quantitative data on a particular phase. Many studies are initiated on the basis that the proposed payload and vehicle characteristics are known and the task is to find out how (or if) the desired mission can be accomplished. Much of this work is security classified and is, therefore, inaccessible. It is the purpose of this thesis to conduct an academic yet practical study of satellite rendezvous so that students may gain an insight into a very complex problem.

An important part of this subject that is specifically not included in this thesis is the topic of guidance and control during the terminal phase of the rendezvous maneuver. This topic appears to be a study in itself inasmuch as the terminal phase shifts to "closed loop" control with feedback from radar as the rendezvous vehicle closes on the target. Some facts about the capabilities of the acquisition system and, in fact, entire specification of the means to be used by the rendezvous vehicle in completing the terminal phase must be outlined to justify the definition

of a successful arrival. Sears and Felleman in reference 2 give an excellent example of this type study. Nason establishes the conceptual limitation of proportional navigation in reference 3.

CHAPTER II

STATEMENT OF THE PROBLEM

General

The earth satellite is a unique device since, unlike vehicles that operate in the earth's atmosphere, its flight is a regular pattern in space determined primarily by the velocity imparted to the vehicle at the beginning of its orbit. While velocities of earth satellites are high compared to any motion within the earth's atmosphere, the motion of the satellite is predicted with accuracy and it is this statement that forms an important base utilized in this thesis.

Satellite rendezvous is the problem of bringing together two objects of relatively small size when the original distance between the two objects and their velocities are both very large by ordinary engineering standards. When the target orbit elements are known, the task of the rendezvous vehicle (RV) is to match them. In practically any circum-

stances, even the simplest case, there exist an infinite number of possibilities for accomplishing a rendezvous.

The general case of matching the ephemeris of an arbitrary target after launch from an arbitrary point in space (surface of the earth) is unmanageable and probably unrealistic. Although an effort is made to keep the problem as general as possible, certain conditions are set before proceeding.

The Specific Problem

The study begins with the assumption that the orbit of the target and the trajectory of the RV are in the same plane. If the orbit of the target has an inclination less than the latitude of the launching point then plane-change is required as the target never passes over the launch point. With angles of inclination greater than this latitude the launch point is in the plane of the target twice each day giving two opportunities for launch into the target plane. Except for polar orbits, one of these opportunities requires a westward launch, an undesirable choice, thus the desirable eastward launch into the target plane is available only once a day.

Now consider further the odds that the target is at a suitable position for rendezvous at the time the launch point passes through the target orbit. Obviously a wait of many days may be required before the correct conditions are met.

A study of the wait time problem is made by Houbolt in reference 4. In this paper he demonstrates for a specific case the limited number of rendezvous opportunities available in a given period and points out the increase in this number given by the "Adjacent Launching Technique". In this scheme the RV is launched into an orbit of the same altitude as the target with the target at a suitable position for rendezvous. When the RV crosses the target orbit it adapts to this new orbit with a plane-change maneuver and begins the terminal phase of the rendezvous.

Inasmuch as changes of orbit inclination are accomplished by an impulse normal to the original velocity vector, the change of plane requirement is in addition to the in-plane requirements and is not considered further.

Since the target is, by definition, the point in space to which we wish to move the RV, it is reasonable to assume that the target has a regular

motion easily predicted. A circular orbit best meets this requirement and this is not an unrealistic assumption; for many reasons an approximately circular orbit is desired for reconnaissance, survey or communications satellites.

Now within a given plane and with a circular target orbit there still remain an unlimited number of trajectories that meet the requirement of placing the RV at the satellite's position in space. These trajectories form the basic conic sections, ellipse, parabola and hyperbola, passing through the two required points. Only a minimum of computation is required to see that hyperbolic and parabolic trajectories have excessive velocity requirements when compared with the elliptical path. The elliptical trajectories that qualify are those with apogee greater than or equal to the altitude of the target. The former, often called a "fast ellipse", crosses the target orbit and this is undesirable for two reasons:

1. To match the orbit of the target the RV requires in-plane normal thrusting at the cross-over point. Normal thrusting is inefficient and is only used as a last resort.

2. Closing velocities at the cross-over are very high and rendezvous requires critical timing.

The ascent ellipses considered in this thesis are those with apogee equal to the altitude of the target. This type trajectory gives a flight path at apogee that is tangent to the orbit of the target.

Within the conditions of being in-plane and tangent at arrival, there is an alternative, four-impulse, "parking orbit" ascent. Briefly, parking orbit ascents consist of a launch into a low altitude orbit, a waiting period until the relative positions of RV and target are correct, then a transfer to the target orbit. The primary advantage of such a scheme is the relaxation of strict requirements for leaving the launching pad at a given time; the low altitude orbit allows the RV to "gain" on the target. The disadvantages of such a system are related to this thesis in that errors in burnout velocity are not only multiplied by the requirement for two "transfer" maneuvers, but initial errors in many cases "grow" during the wait period. These and other factors are considered in detail by Duke, Goldberg and Pfeffer in reference 5.

Coasting in free flight from burnout of the booster stages until the RV is at the altitude of the target is termed a "direct ascent". Such a trajectory has a minimum velocity requirement in the classical Hohmann maneuver and an upper limit in the vertical ascent. This thesis will be concerned with these two-impulse type rendezvous trajectories.

To review, then, the specific problem investigated in this thesis is an in-plane rendezvous with a target in circular orbit utilizing a direct ascent elliptical trajectory that is tangent to the target orbit.

Basic Assumptions

The earth is assumed to be a homogeneous sphere. The earth is, of course, quite irregular; it is more an ellipsoid than a sphere.

This oblateness has three main perturbing effects upon the orbit of an earth satellite, all dependent upon altitude and inclination of the orbit:

1. Regression of the nodes. This is motion of the orbital plane about the earth's axis in a direction opposite to that of the satellite.

2. Precession of the line of apsides. This motion is an angular rotation of the axis of the orbit in inertial space. It may be in the same or the opposite direction of the satellite motion depending upon the amount of inclination.

3. The oblateness of the earth also causes slight fluctuations of the radius (altitude) and period of the satellite.

All of the above effects are important in sustained satellite operations and their effects on orbits adds to the knowledge of the shape of the earth, however, such effects are of small magnitude in direct ascent rendezvous trajectories.

As an example, in reference 6 the formula:

$$\Delta = 2\pi(1.637 \times 10^{-3}) \frac{R_e^2}{(R_e + z_s)^2} \cos i$$

is given as the measure of angular motion of the nodes in radians per revolution of the satellite. For an altitude of 1000 n. miles and with a circular orbit inclined at an arbitrary angle of 45° , the regression of the nodes is 4.38×10^{-3} radians for every revolution of the target. The ascent trajectories described in this thesis are of short duration compared with the period of the satellite

target and only a portion of this regression of the nodes takes place while the RV is in flight. For the 1000 n. mile target the ascent time varies from 3134 seconds to 843 seconds or 11% to 40% of the period of the target. Thus the regression of the nodes is from 13,000 to 47,000 feet at this geocentric radius.

This example is given only to show the order of magnitude of the motion in this case. In the rendezvous situation the perturbations affect the RV as well as the target at the same time and in the same direction. The error is, then, of differential order when compared with the above.

If these effects are expressed as functions of position they can be introduced into the integration of the equations of motion for the rendezvous computations, but they are not the subject of the present investigation and can influence it only through higher order effects.

To continue with the basic assumptions, booster burnout altitude is taken as sixty nautical miles for all cases. Without a detailed examination of the booster performance and maneuvering scheme it cannot be determined whether this altitude is appropriate for a given trajectory, but some arbi-

trary altitude is needed for a common base from which to compare the coasting performance. This altitude is assumed in numerous references such as Eggleston in reference 7.

It is further assumed that the ephemeris of the target is known exactly. The technique of tracking an earth satellite and determining its ephemeris is fairly well formalized and high speed digital computer routines are available for this purpose. The technique is described by Sandorff in reference 8. For this study, however, uncertainties of the satellite position are simply uncertainties in the origin of the final error coordinates. While this may be an important consideration, it is independent and can be introduced as necessary in a system study.

Although the ability to predict launch time continually improves with experience in missile operations, there are still uncertainties in time of lift-off as well as error build-up during burning. The time of flight or coasting time varies considerably with the particular trajectory selected (as seen below) giving some flexibility in matching time. A pre-launch computer keeps track of this time and, over some useful range

determined by other considerations, selects the exact trajectory to match time. In a similar manner, during the burning period performance is monitored and cut-off corrections computed to match the time. In very general terms, Duke, Goldberg and Pfeffer in reference 5 take notice of this type error but its implementation depends upon system specifications. If an error in time alone is accepted and no compensation is made then the arrival error is easily computed from the satellite velocity. To get to the point of the study, it is assumed further that booster burnout is compelled to occur exactly on time.

Uncertainties Considered

The uncertainties considered in this thesis are those in the booster burnout velocity vector. These errors are described as:

1. Position errors

- a. ahead (RANGE error)
- b. above (ALTITUDE error)
- c. to one side (TRACK error)

2. Velocity errors

- a. error in magnitude

b. error in direction

(1) deflected above or below
the ideal

(2) deflected to one side of
the ideal

The exact meaning of "position errors" depends upon the type of guidance equipment utilized by the booster stage; the full implication of this point is carefully discussed in Chapter III.

CHAPTER III

COMPUTATIONAL METHODS AND PROCEDURES

Analytical Computation of Reference Trajectories

The elements that describe the target orbit and the family of possible direct ascent ellipses tangent to it are defined with the help of Fig. 1 below.

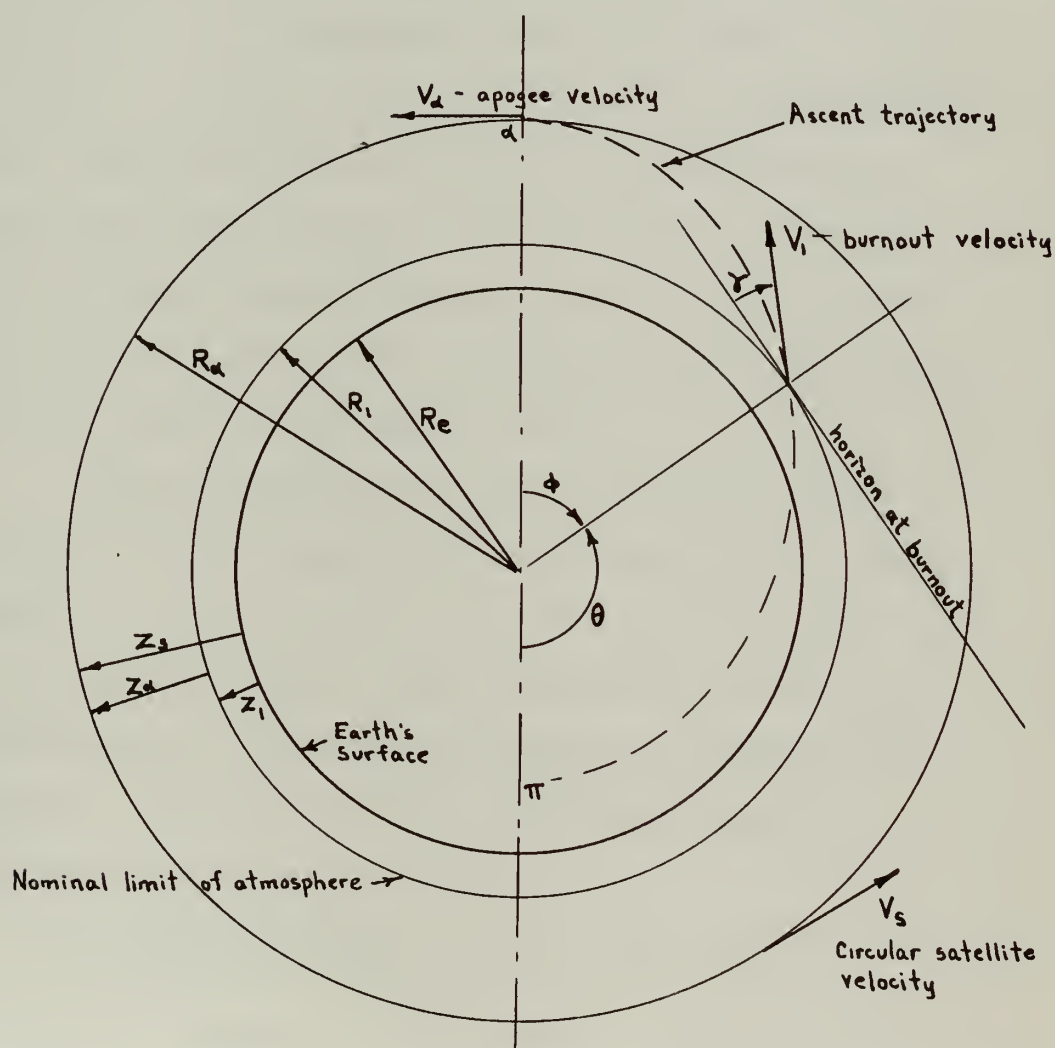


Fig. 1. Definition of Elements Used in Computational Procedures.

From the figure it is seen that selecting a target altitude fixes R_α for all ascent ellipses, since the rendezvous condition is that apogee occurs at target altitude. An additional variable is needed to specify a particular trajectory from the possible family of ellipses, and the range angle, ϕ , best describes what is a long or short ascent. Although these two variables are selected for classification of results, the conventional two-body equations are more familiar in terms of the standard parameters e , a and θ . The desired values are computed by successive substitution in to the numbered equations below.

These equations are not involved, but they require careful numerical evaluation for accurate results. With dimensions in millions of feet, the IBM 650 carrying eight significant digits is adequate for computation directly in engineering units. The three physical constants for the numerical work are:

G_m (gravitational constant times mass of the earth)

$$1.40768 \times 10^{16} \text{ ft}^3/\text{sec}^2 \quad \text{Ref. 8}$$

R_e (radius of a sphere with volume equivalent to that of the earth)

$$3437.866 \text{ nautical miles} \quad \text{Ref. 9}$$

International nautical mile = 6076.1155 feet

Ref. 10

Altitude traversed in ascent

$$z_{\alpha} = z_s - z_1 \quad (3.1)$$

Minimum eccentricity

$$e_{\min} = \frac{z_{\alpha}}{2R_1 + z_{\alpha}} \quad (3.2)$$

Angle from perigee

$$\theta = (180^{\circ} - \phi) \quad (3.3)$$

Eccentricity

$$e = - \frac{\cos\theta + 1}{2 \left[\cos\theta + 1 + \frac{z_{\alpha}}{R_1} \right]} + \sqrt{\frac{1}{4} \left[\frac{\cos\theta + 1}{\cos\theta + 1 + \frac{z_{\alpha}}{R_1}} \right]^2 + \left[\frac{z_{\alpha}/R_1}{\cos\theta + 1 + \frac{z_{\alpha}}{R_1}} \right]} \quad (3.4)$$

Satellite velocity

$$V_s = \sqrt{\frac{Gm}{R_1 + z_{\alpha}}} \quad (3.5)$$

Booster burnout velocity

$$V_1 = V_s \sqrt{\frac{2 z_{\alpha}}{R_1} + 1 - e} \quad (3.6)$$

Apogee velocity

$$V_{\alpha} = V_s \sqrt{1 - e} \quad (3.7)$$

Injection velocity increment

$$V_2 = V_s - V_\alpha \quad (3.8)$$

Semi-major axis

$$a = \frac{R_1 + z_\alpha}{1 + e} \quad (3.9)$$

Booster burnout attitude angle

$$\gamma = \arctan \left[\frac{e \sin \theta}{1 + e \cos \theta} \right] \quad (3.10)$$

Time to apogee

$$T = a \sqrt{\frac{a}{Gm}} \left\{ \pi - \arccos \left(\frac{a - R_1}{ae} \right) + e \sin \left[\arccos \left(\frac{a - R_1}{ae} \right) \right] \right\} \quad (3.11)$$

This information, computed for 12 target altitudes from 200 to 20,000 nautical miles and range angles of 180, 90, 45 and 22.5 degrees permits selection of a smaller representative sample for detailed error analysis. Target altitudes of 400, 1,000, 5,000 and 20,000 n. miles are used in the error analysis to follow.

The Error Analysis

A complete error analysis of even this basic set of trajectories by analytical methods, if possible at all, is certainly complex because several orbital elements are often changed by a

single error at booster burnout. For instance, the variation in semi-major axis can be found as a function of a change in γ , but before an error can be isolated, the corresponding change in eccentricity and the shift of the line of apsides must be determined. Finally, arbitrary position as a function of time is transcendental.

Braham and Skidmore in Ref. 11 give analytical expressions for changes in altitude, final velocity and final flight path angle as a function of certain initial condition errors. These expressions are cumbersome. The one for final flight path angle requires evaluating six lengthy expressions to get the necessary partial derivatives, which in turn require evaluating four other equations equally as long to solve for some of the coefficients. Even if a complete set were available, evaluation in any quantity calls for a machine computer program.

Such analytical solutions are interesting from the theoretical point of view, but it is felt that the most practical way to get a complete error analysis is by numerical integration of the equations of motion.

The motion of a body subject to an acceleration is described by the vector equations:

$$\bar{V} = \int \bar{a} \, dt + \bar{V}_0 \quad (3.12)$$

$$\bar{R} = \int \bar{V} \, dt + \bar{R}_0 \quad (3.13)$$

In coasting flight near the earth, the only acceleration is that due to the gravitational field, $\bar{G}$, which is a function of $\bar{R}$. Thus, for step-by-step integration, the two simultaneous equations are actually solved in order as follows:

$$\bar{a} = \bar{G} = \left[\frac{Gm}{R^3} \right] \bar{R} \quad (3.14)$$

This acceleration is expressed in components in whatever coordinates $\bar{V}$ is given, and each component of $\bar{V}$ and $\bar{R}$ is computed as:

$$V_1' = G_1 \, \Delta t + V_{10} \quad (3.15)$$

$$R_1 = V_1 \, \Delta t + R_{10} \quad (3.16)$$

The resulting $\bar{R}$ is used to compute a new $\bar{G}$; the resulting $\bar{V}$ is the $\bar{V}_0$ for the next step, and the resulting $\bar{R}$ is also the $\bar{R}_0$ for the next step. This procedure is repeated up to apogee time.

A complete program with this capability, developed by Mr. Robert Brown of the M.I.T. Instru-

mentation Laboratory and adapted to this problem, is used in the error analysis. With this program and the IBM 709 computer at the M.I.T. Computation Center, trajectories for four altitudes and four range angles are thoroughly investigated.

The method is straightforward and accurate when used with a suitable integrating time step. The validity can be checked for each trajectory by integrating over the complete time of flight with no errors introduced and comparing the apogee conditions with the analytical results from the equations above. As an example of the accuracy attained the zero-error numerical integration for ascent to a 5,000 n. mile altitude at a range angle of 90° using a 50 second time step agreed to within 80 feet ALTITUDE and 7 feet RANGE distance - this is at a geocentric radius of over 51 million feet.

The program uses spherical coordinates in the actual integration, but keeps track of the components in terms of an orthogonal frame called a platform frame. This frame is introduced to simplify the presentation of the error components, and has a positive X axis in the RANGE direction

along the horizontal in the direction of flight, a positive Z axis upward along the vertical ALTITUDE direction, and a positive Y axis to the right looking in the direction of flight.

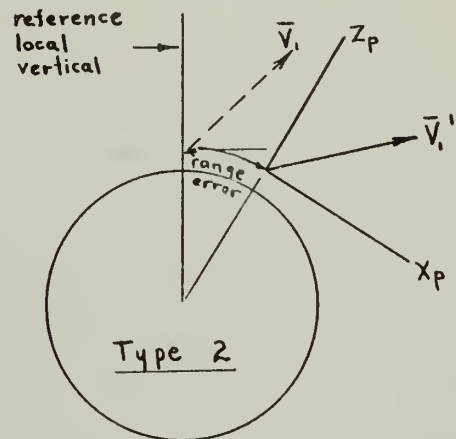
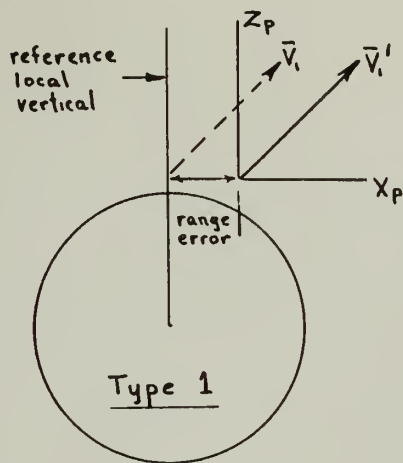
A platform frame is specified because for inertially guided vehicles directions are referred to a stabilized platform. The platform is non-rotating in earth-centered inertial coordinates (type 1) or its Z axis tracks the local vertical (type 2). The effect of position error on both types is shown in Fig. 2.

The first type seems most likely for this application, but it is used here for a more basic reason. With error effects available for the first type, they can be computed for the second type. Note from the figure that ALTITUDE errors alone are the same in either system, but that with RANGE or TRACK error, the relative alignment of the G vector depends on the type system assumed. It is seen though, that a type 2 RANGE error consists of a type 1 RANGE error plus a velocity misalignment ($\delta\gamma$) which is easily computed from booster burnout radius and the RANGE error.

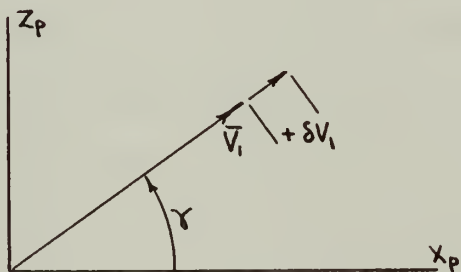
Effects due to the actual change in height caused by a type 1 RANGE or TRACK error are negligible because, for reasonable errors, this change

in height is very small, and the corresponding position error derivatives are also very small. The same is not true for misalignment errors. The errors may be fractions of a milliradian, but this kind of error has a large end effect.

Finally, for TRACK error, the motion due to the type 1 position error must be obtained by numerical integration, but for the type 2 platform where the ALTITUDE component of velocity is aligned with the local vertical, the entire motion lies in the plane of the error trajectory. When a trajectory lies entirely in a plane and the inclination with respect to another plane is known, the lateral displacement is just a function of the geometry. This point is brought out in more detail in Chapter IV.

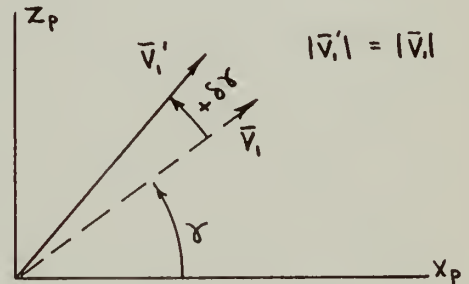


Comparison of two types of platform coordinate frames. Angle for reasonable position error greatly exaggerated.



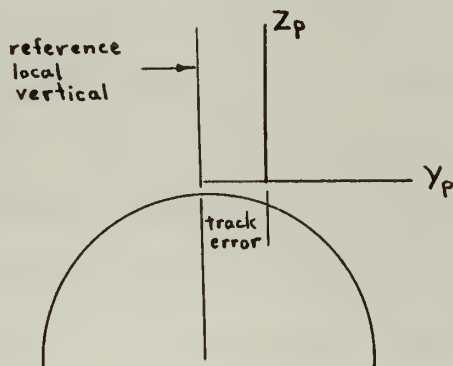
Ref. local horizontal

Error in magnitude of booster burnout velocity.

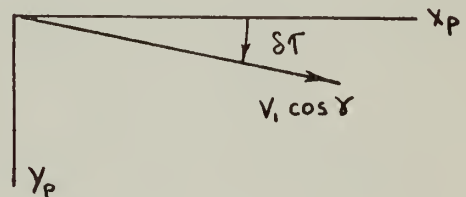


Ref. local horizontal

In-plane misalignment of booster burnout velocity.



Track Error - Type 1 system as used herein.



Lateral Misalignment of booster burnout velocities.

Fig. 2. Platform Coordinates and Definitions of Booster Burnout Errors.

CHAPTER IV

RESULTS

Summation of Velocities

The summation of booster burnout velocity and injection velocity increment (ΣV) for various altitudes and range angles is shown in Fig. 3. It must be borne in mind that the values represented here are the theoretical minima; they do not reflect the additional velocity that must be provided to overcome drag and gravity losses during accelerations to V_1 .

What is considered important about these plots is the effect of altitude and range angle on ΣV . Note that while analysis always prescribes the Hohmann maneuver for minimum ΣV , this is not necessarily the primary consideration in a practical case. For instance, with a 400 n. mile orbit altitude the range angle is reduced to 55° before ΣV increases 1% over the Hohmann value; it is reduced to 18° before ΣV increases 5%. Similarly for a

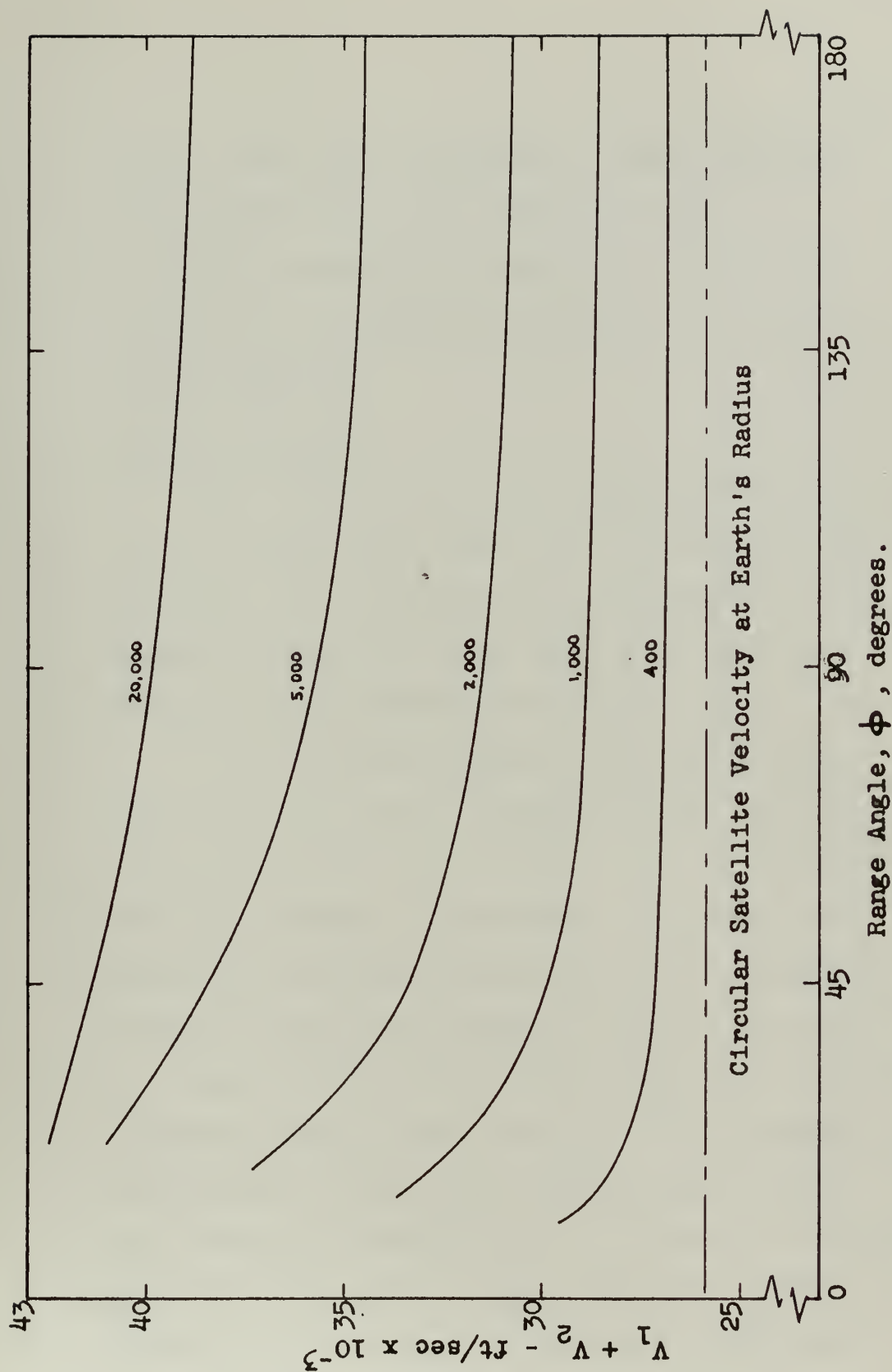


Fig. 3. Summation of Booster Burnout Velocity and Injection Velocity Increment. Altitudes in nautical miles are shown adjacent to the curves.

1000 n. mile orbit altitude the range angle is down to 40° before ΣV increases by 5%.

These examples are cited only to call attention to what may be one corner of a many sided compromise.

Velocity Division

Fig. 4 is a plot of the separate components of ΣV versus range angle for a number of orbit altitudes. The information is displayed on two separate plots for low and high altitudes to eliminate the clutter of several overlaps.

V_1 , the booster burnout velocity, is the velocity which must be imparted to the RV to send it coasting up to apogee. V_2 , the injection velocity increment, represents that propulsive capability that must be carried to the rendezvous point in order that satellite velocity be matched.

From another point of view, V_2 is the relative velocity between target and RV. If the ascent trajectory is perfect and time matched exactly, the injection thrusting can be initiated by elapsed time, but with error buildup during the ascent it seems most likely that the terminal phase will be

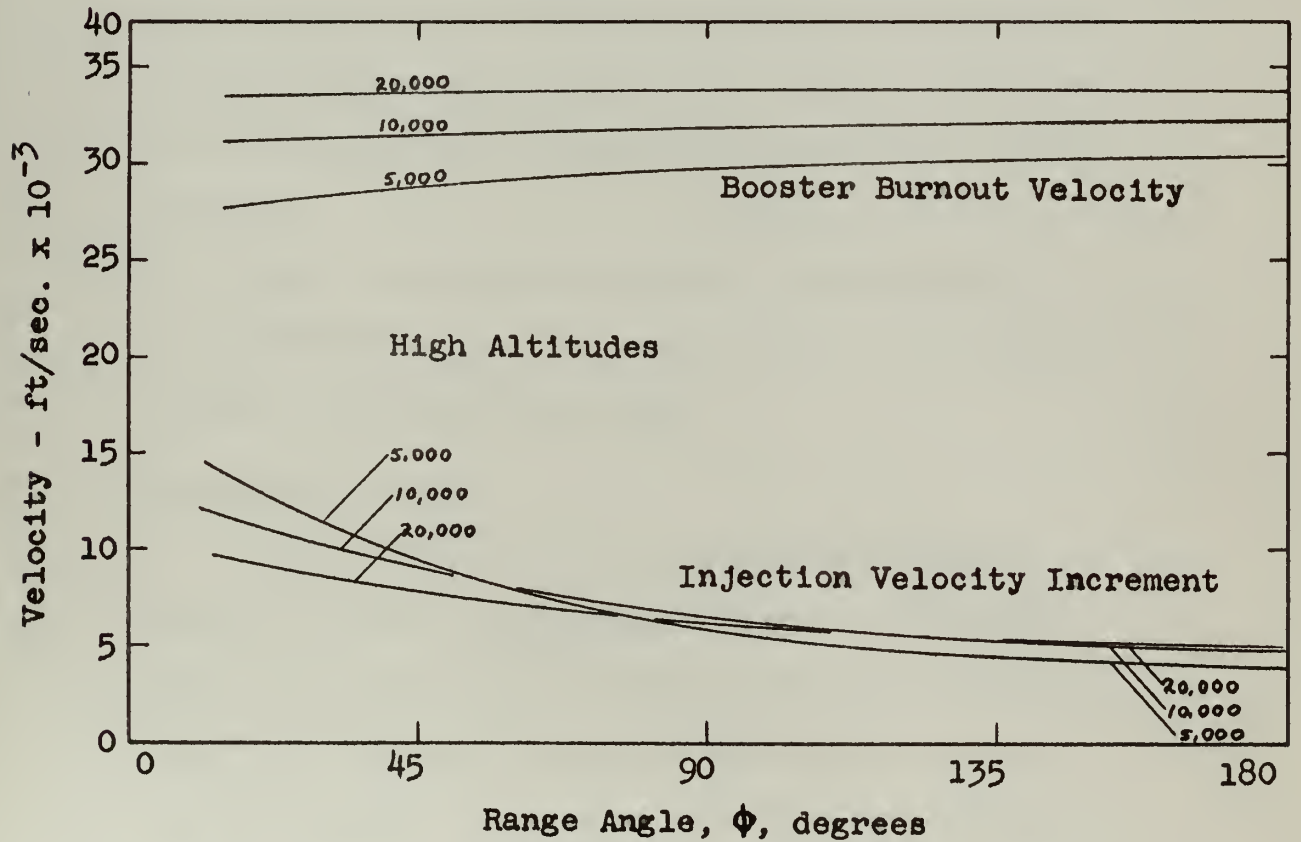
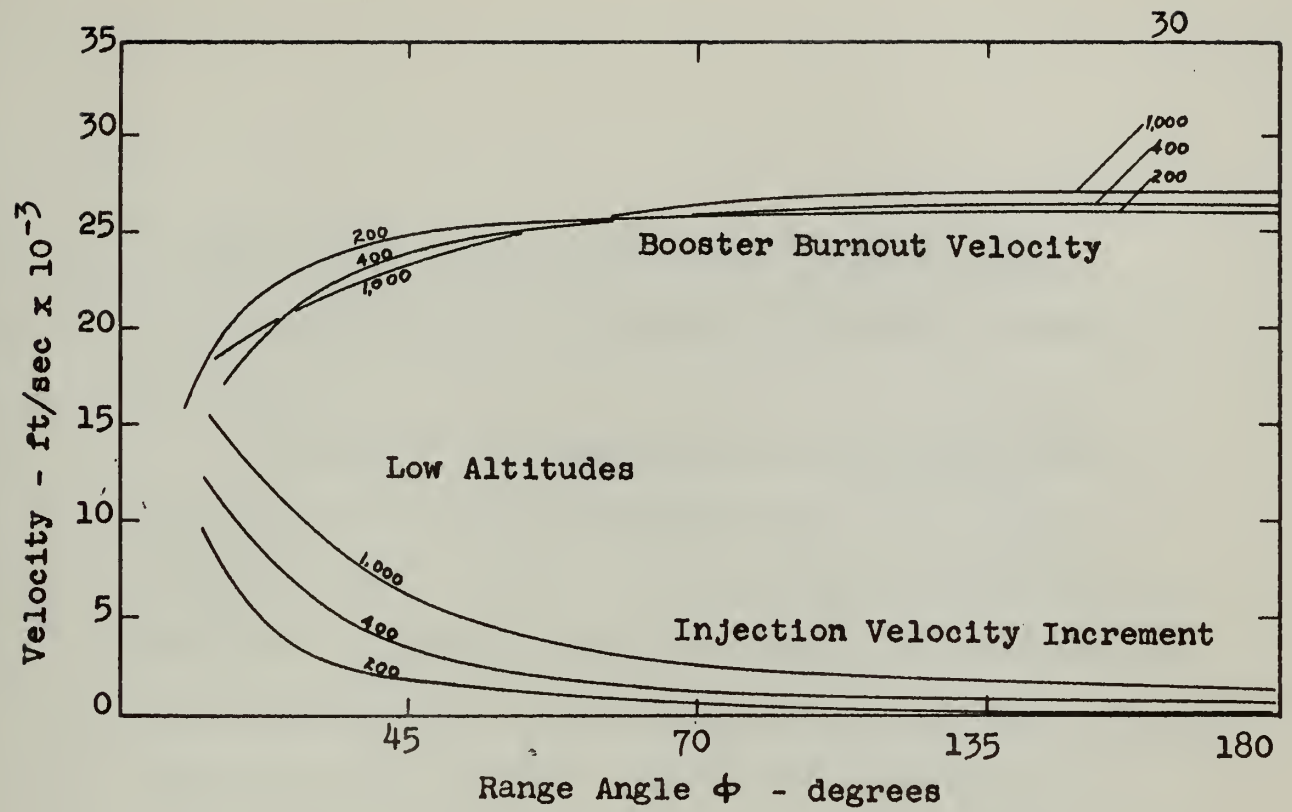


Fig. 4. Effect of Altitude and Range Angle on Booster Burnout Velocity and Injection Velocity Increment.

closed-loop. Thus, this relative velocity may be important during the acquisition of the target by the RV.

As before, for computation of propulsive requirements the drag and gravitation losses must be added to V_1 . Then, once an effective exhaust velocity or specific impulse is known or assumed, quick preliminary calculations can be made concerning vehicle design. These might consist of selecting an altitude and/or range angle within the capability of a certain system, or, in determining what configuration and staging or what restart capability is needed to accomplish a given mission.

This information forms the second part of the propulsive picture, and another side of the overall compromise involved.

Error Derivatives

The results of the error analysis are reduced to partial derivatives since it is shown that the variation of a final component with respect to a change in the initial value of any other component is linear over what is considered to be a generous range of variation. This direct

proportionality is verified for the 1000 mile orbit for speed errors up to 60 feet per second, for angular deviations of the velocity up to two milliradians and for position errors up to 12,000 feet in any direction. As shown for the errors in booster burnout velocity, plotted on Fig. 5, there is no tendency to depart from linearity at 60 feet per second. Because the functions being considered are smooth and continuous, they do not bend abruptly so it is considered that booster burnout velocity errors up to 100 feet per second and angular deviations involving components of that size could be entertained with confidence. It does not seem likely that such large errors are realistic, although what is tolerable certainly depends on what is said about the terminal system capabilities. In any case, the additional computer time required to investigate by this method and establish the absolute limit of applicability is not justifiable with the many other items being studied with the computer.

The types of booster burnout uncertainties described above are classifiable as in-plane and lateral errors. With three components each to

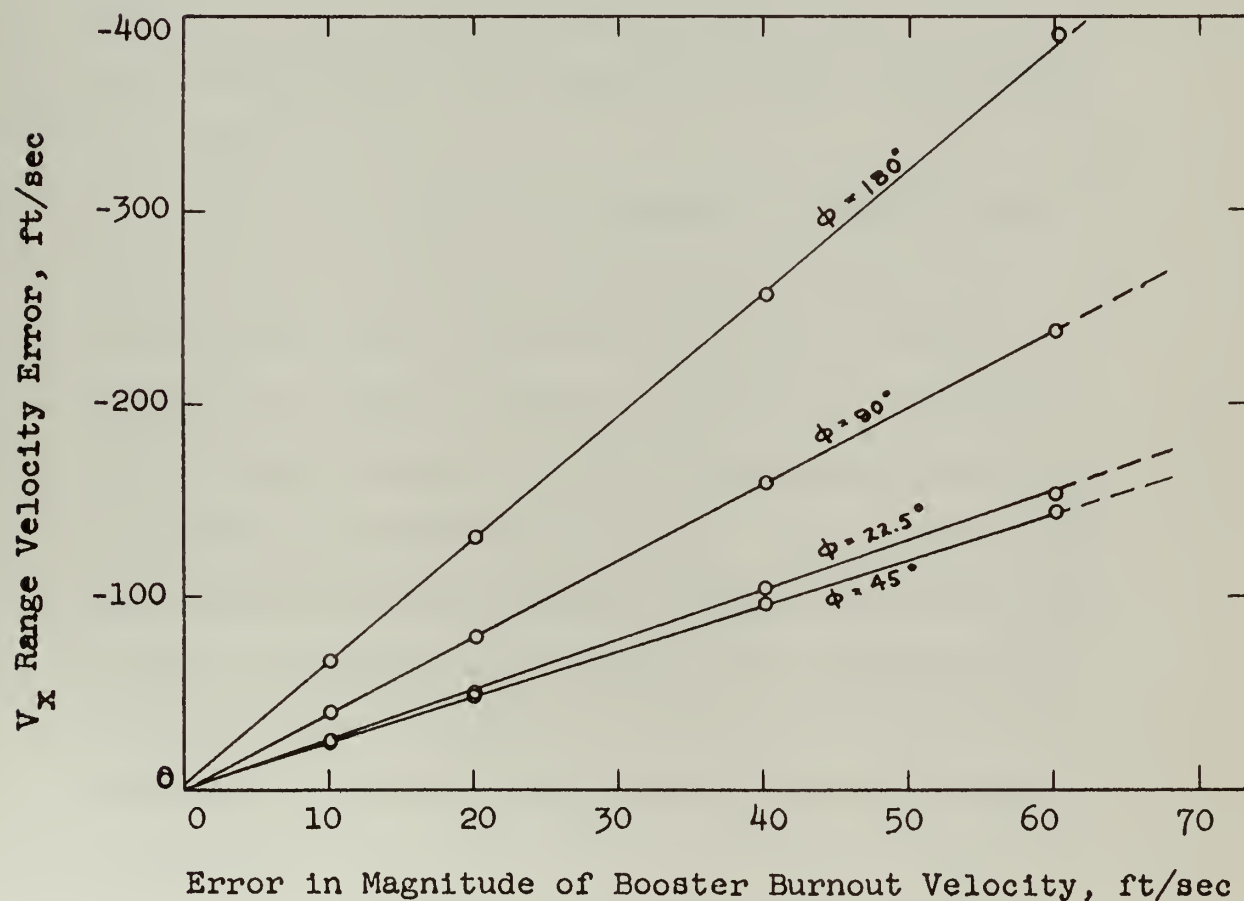
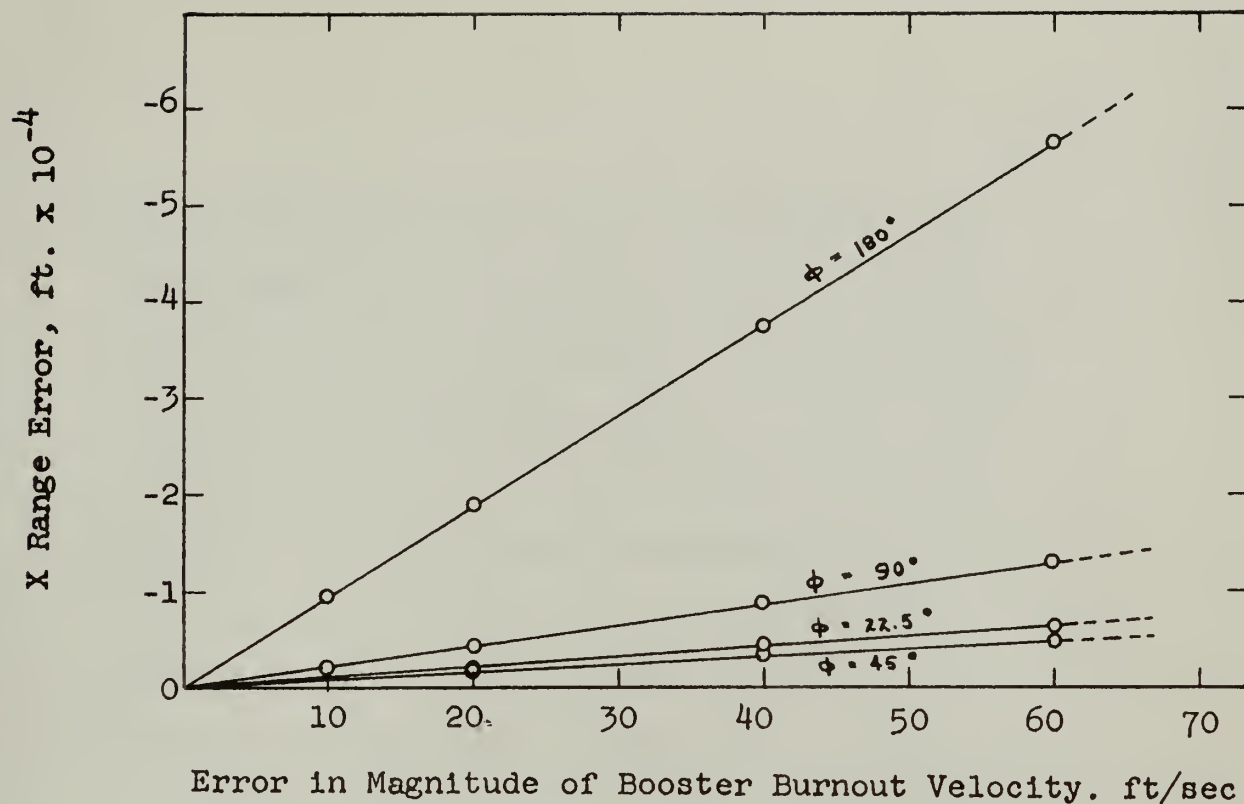


Fig. 5. Effect of Errors in the Magnitude of the Booster Burnout Velocity on Range and Range Velocity Demonstrating Linearity.

define the position and velocity vectors, six differentials are sufficient to describe any possible error in the booster burnout condition. To express the reference apogee or final value of each of the six vector components in terms of each of the six initial components leads to 36 partial derivatives. There is, however, clearly no way for the in-plane effects to induce any lateral perturbations and, as expected, the lateral errors introduce only infinitesimal effects on the final in-plane performance. With this independence, the number of error derivatives is reduced to 20.

While it seems more natural to describe the initial velocity errors in terms of variations in magnitude and direction, the final velocity errors are given in terms of orthogonal components. This is done primarily for ease of combination of components as demonstrated later. Also, since both the target and the reference apogee point have zero Y and Z components of velocity, the computed error components are the actual relative velocity in the ALTITUDE and TRACK directions as viewed from the satellite.

The X component of the error velocity, while

generally quite small compared to either the actual or relative velocities in the RANGE direction, is still important. It is a part of the terminal problem, and the rendezvous is not complete until all components of relative velocity have been exactly matched.

The derivatives relating the initial and final variations are made up of combinations of the following:

1. In-plane - Initial Position (x and z) and velocity (v_1 and γ)

Final position (X and Z) and velocity (V_X and V_Z).

2. Lateral - Initial Position (y) and velocity (τ).

Final position (Y) and velocity (V_Y).

The general form of the derivative as used for evaluating ascent errors is $\frac{\partial(\text{final})}{\partial(\text{initial})}$. This does not preclude using the inverse. If, for instance, it is desired to define allowable final error first, then the permissible booster burnout deviations of a type could be found. However, there are, in general, many combinations of booster burnout errors that can combine to cause similar final errors.

The derivatives are plotted in Figs. 6 through 15 with range angle as the primary variable and the target altitude as the step parameter. Each is explicitly defined on the respective plot with units in feet, feet per second and milliradians.

It is important to note that as range angle approaches zero, the eccentricity of the trajectory ellipse approaches unity and the regular forms of the equations become very sensitive. Judging from a few of the cases for which the vertical ascent may be evaluated, it appears that the error functions in fact are quite unstable at low range angles. For this reason, none of the results should be extrapolated below the 22.5° range angle where the error computations were cut off. There seem to be practical reasons for avoiding very steep ascents such as the high relative velocities, highest possible ΣV , and need for rapid change of attitude in the terminal phase. For such reasons, it is considered that the lack of coverage at these short angles is not important.

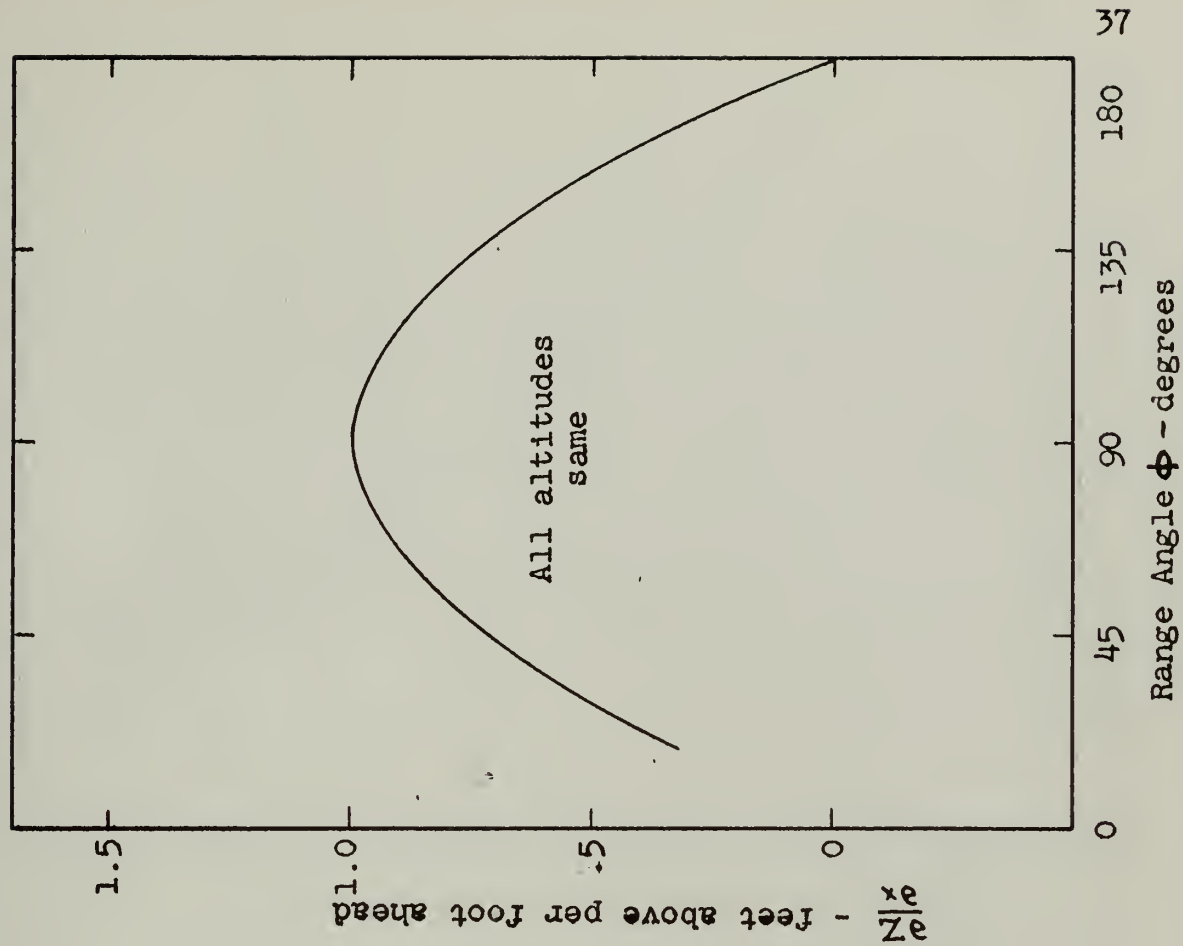
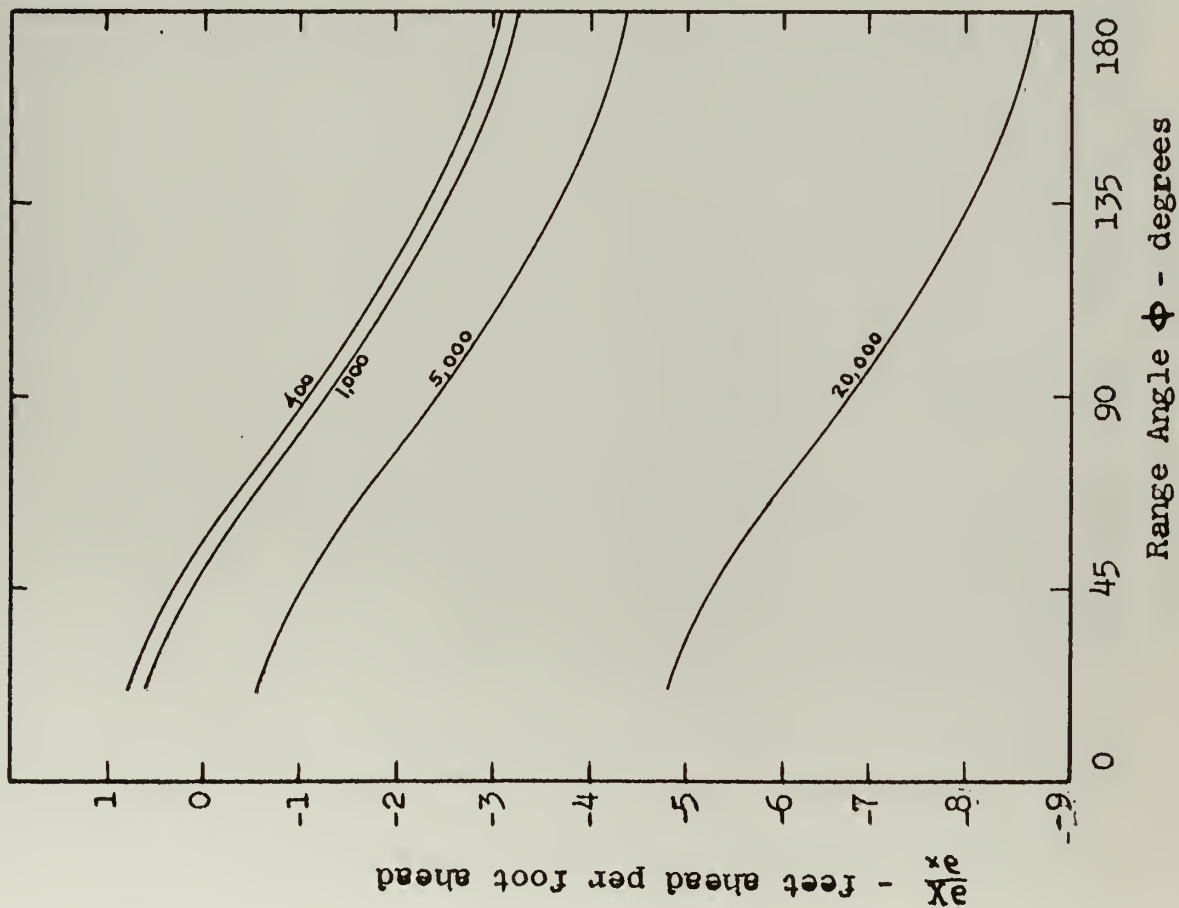


Fig. 6. Effects of Range Error at Booster Burnout on Position at Zero-error Apogee Time

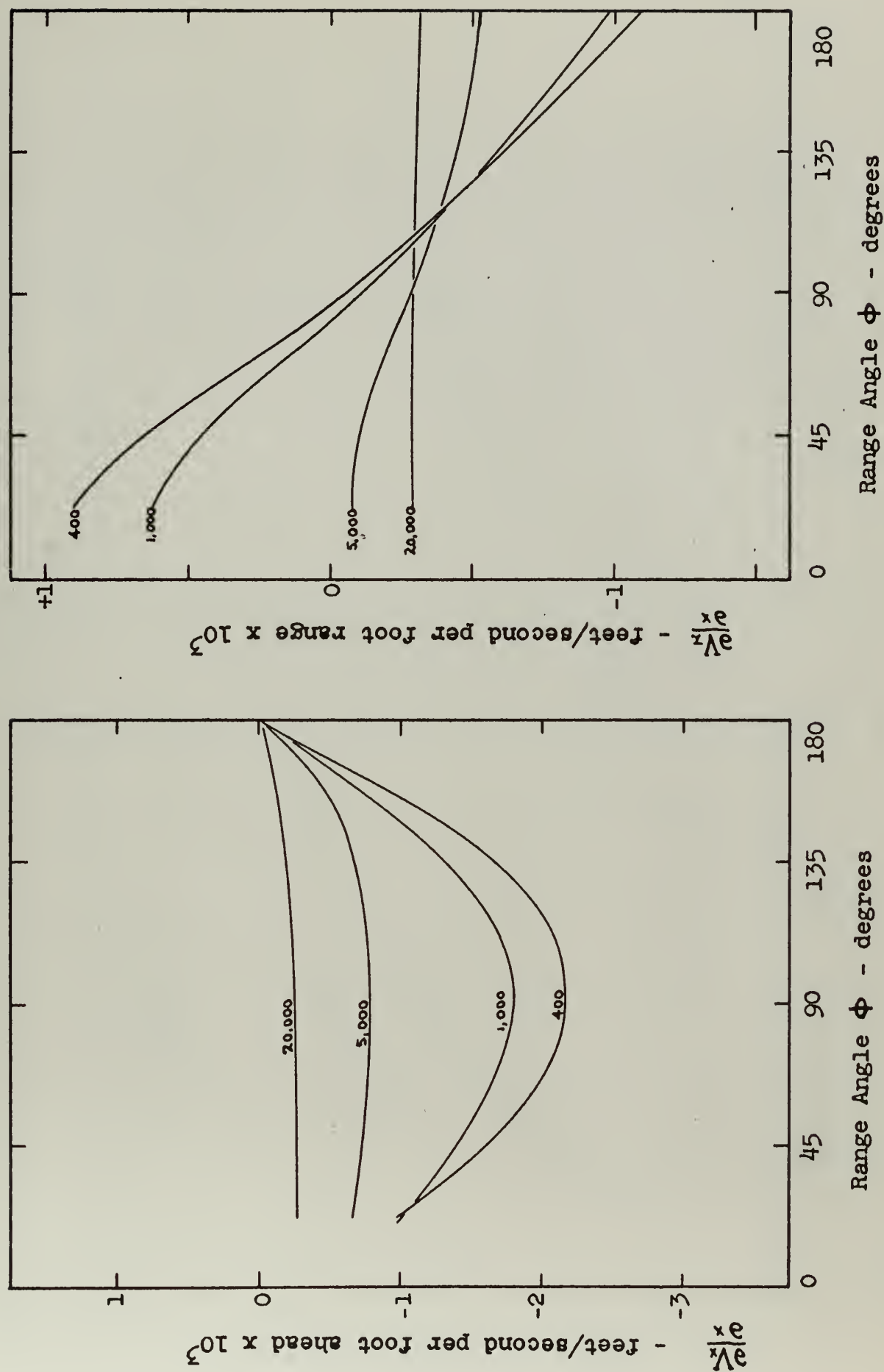


Fig. 7. Effects of Range Error at Booster Burnout on Velocity at Zero-error Apogee Time

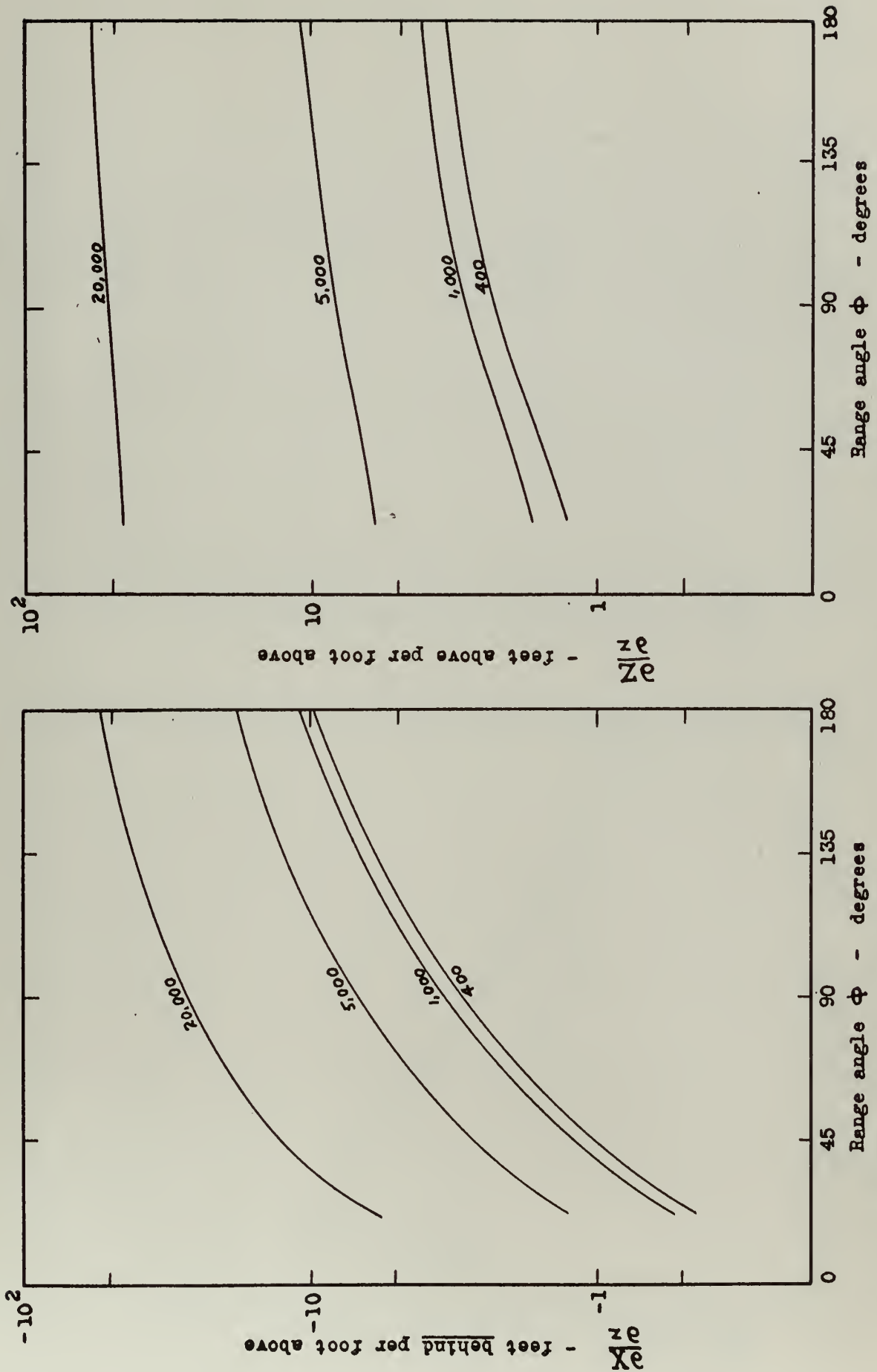


Fig. 8. Effects of Altitude Error at Booster Burnout on Position at Zero-error Apogee Time

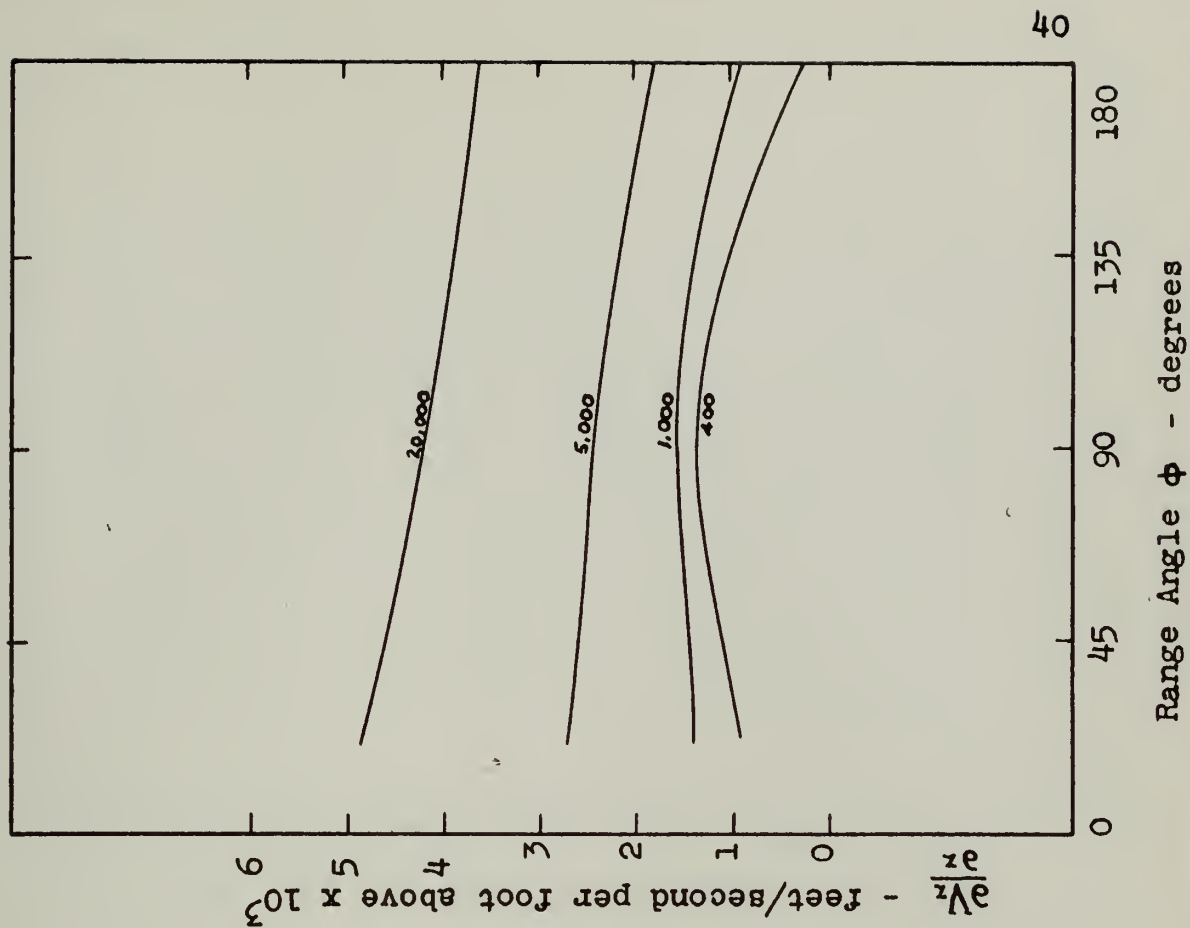
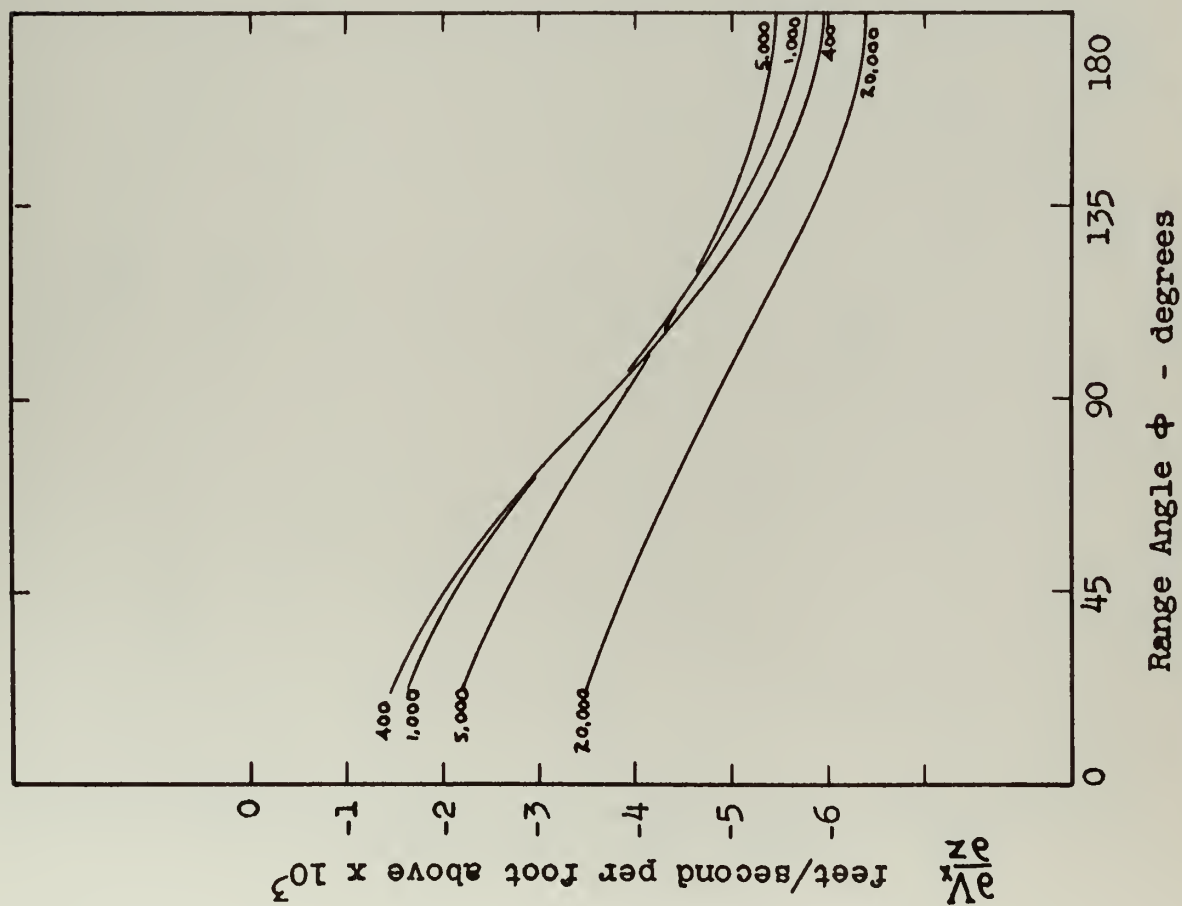


Fig. 9. Effects of Altitude Error at Booster Burnout on Velocity Burnout at Zero-error Apogee Time

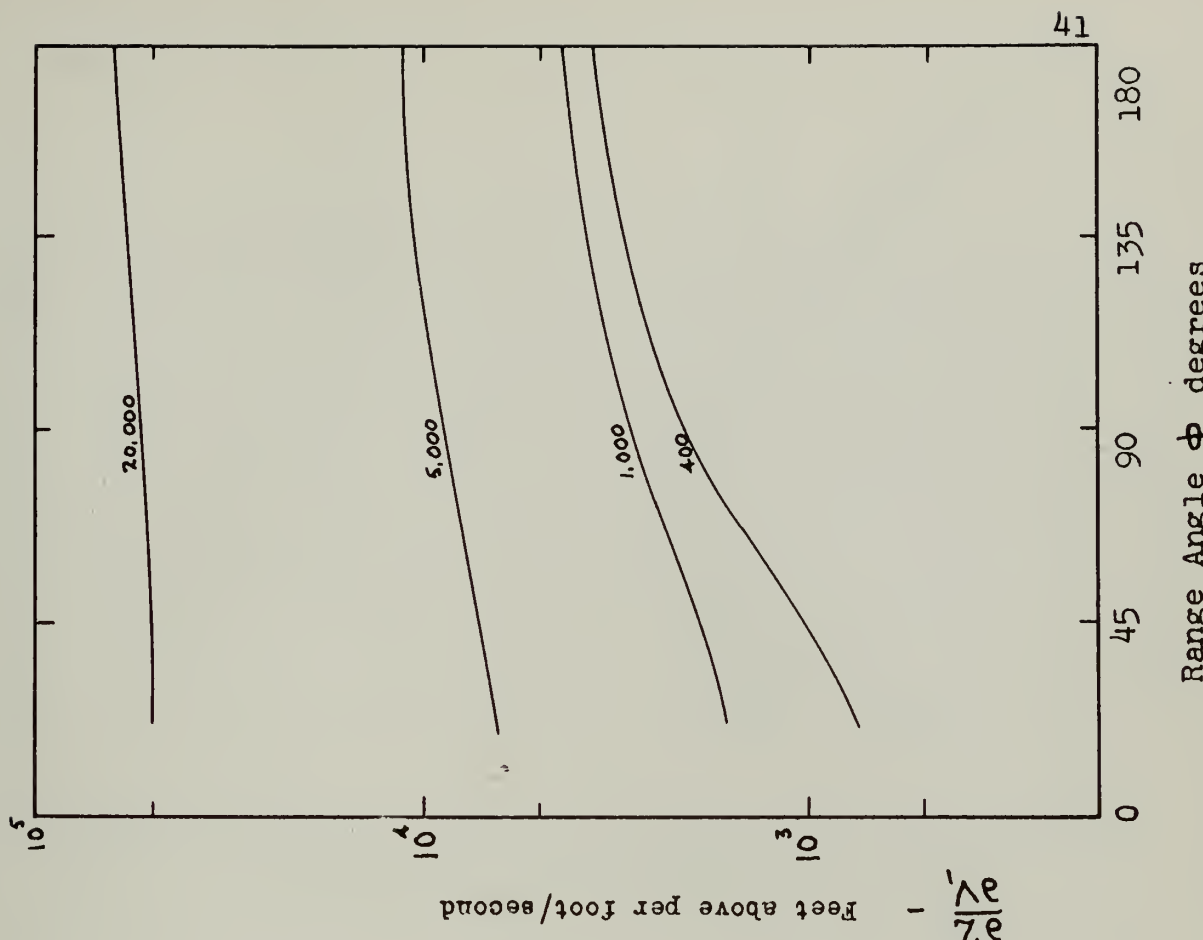
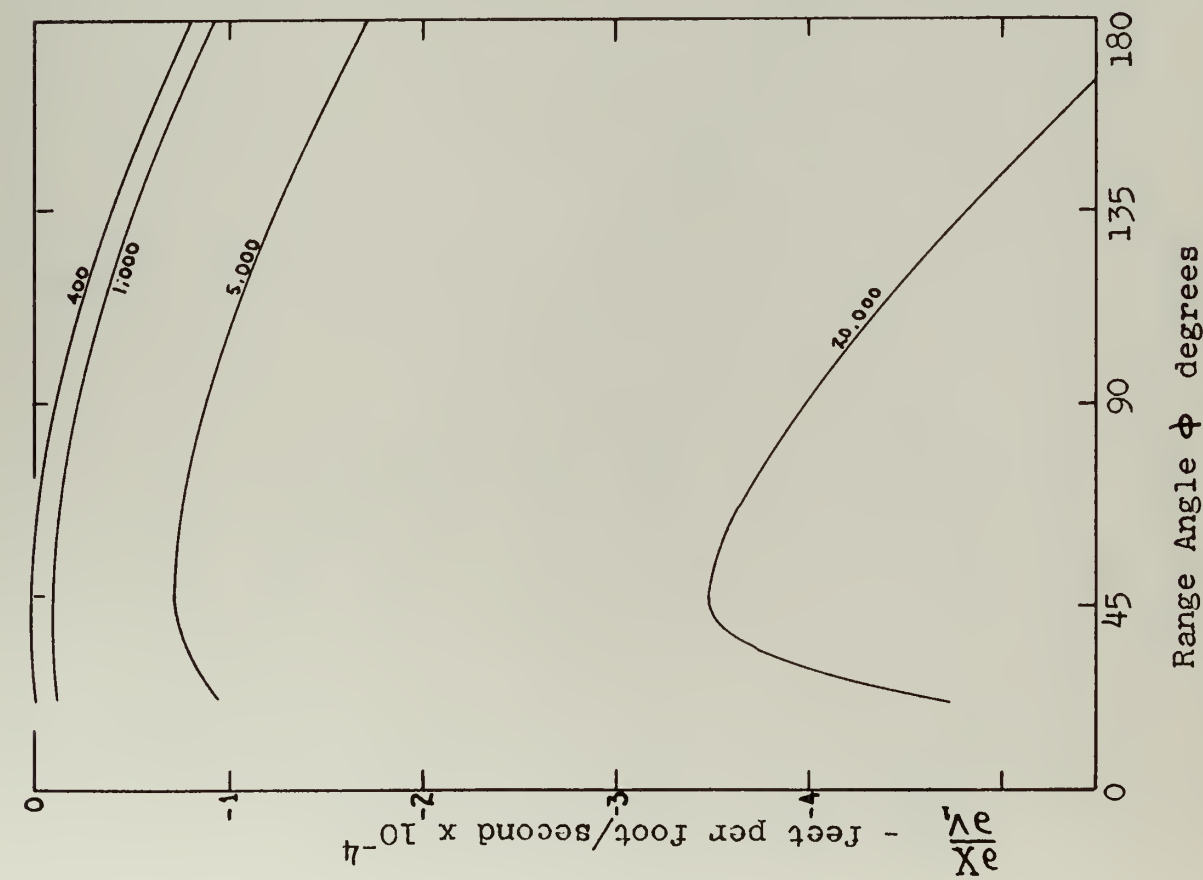


Fig. 10. Effects of Error in Magnitude of Booster Burnout Velocity on Position at Zero-error Apogee Time.

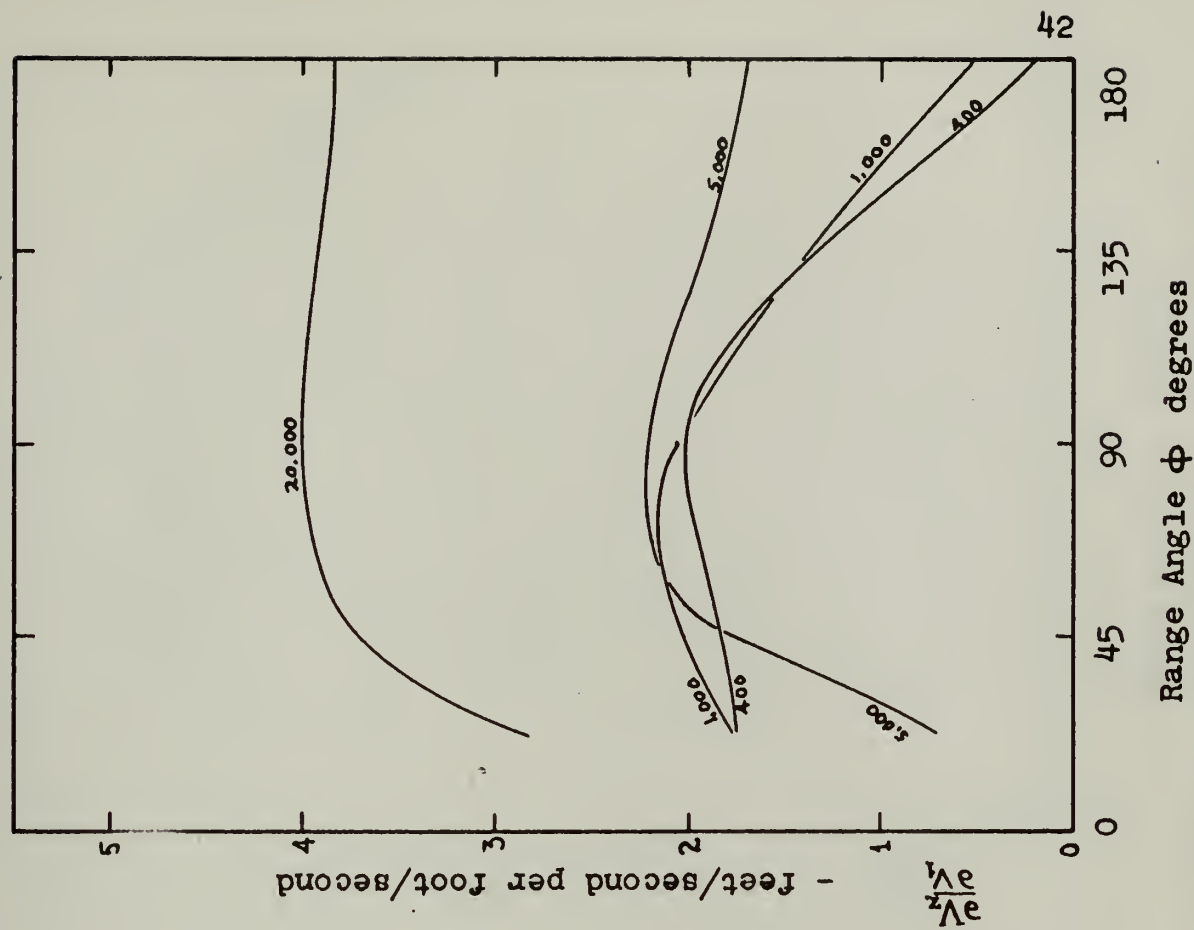
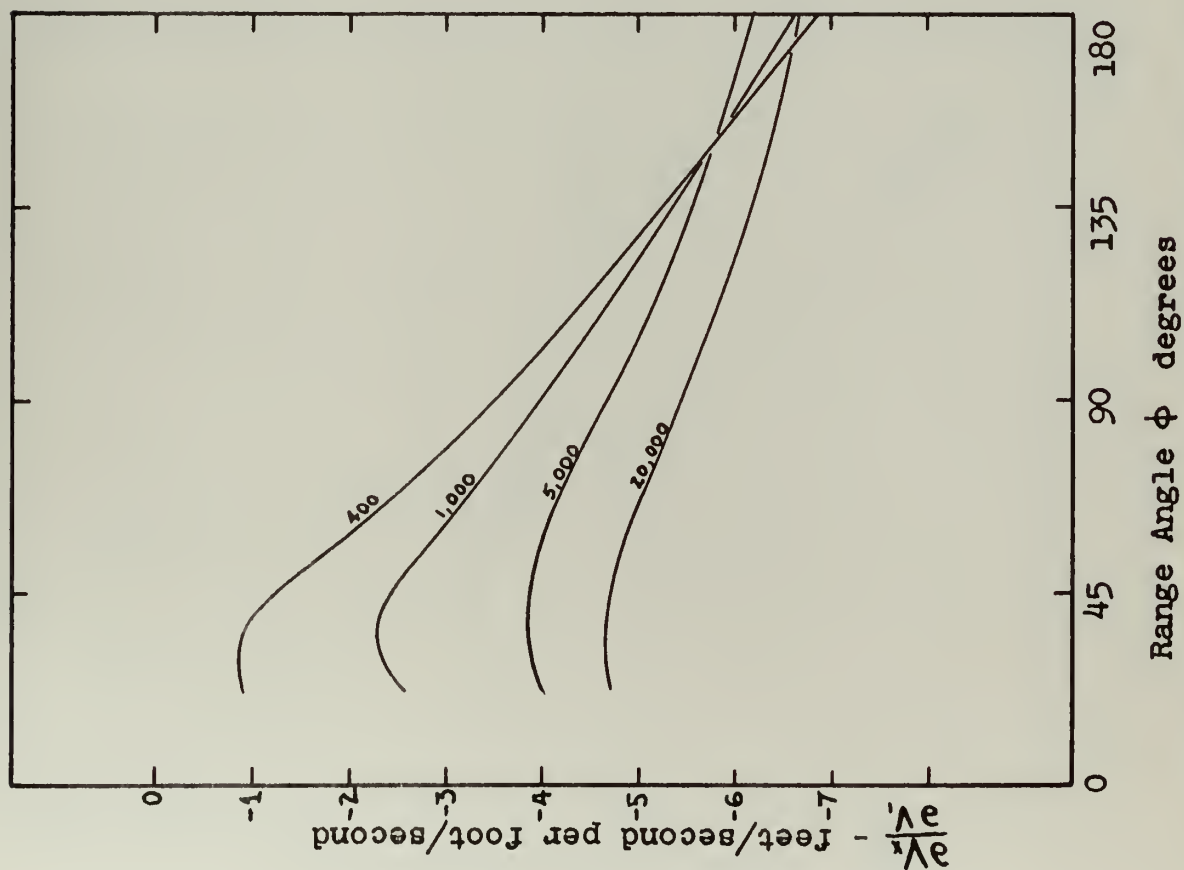


Fig. 11. Effects of Errors in Magnitude of Booster Burnout on Velocity at Zero-error Apogee Time.

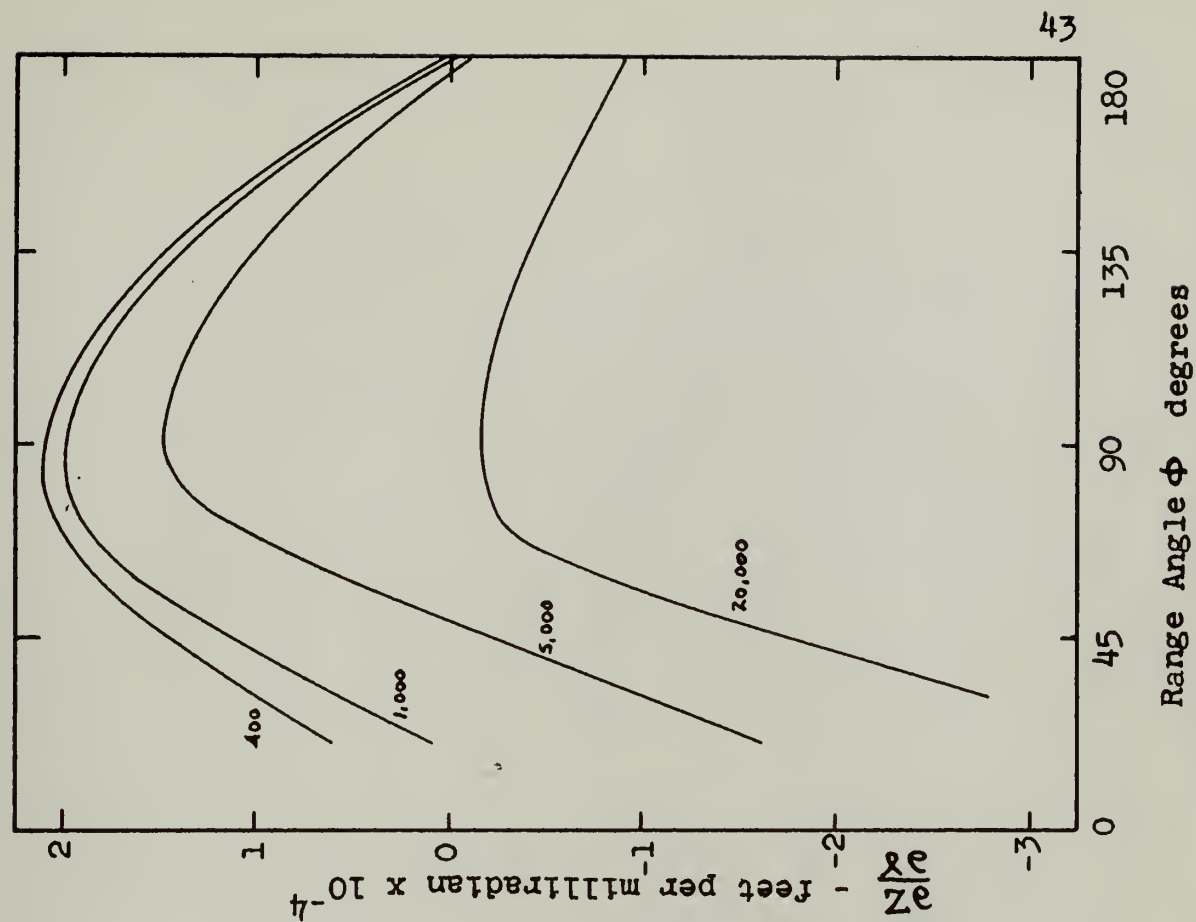
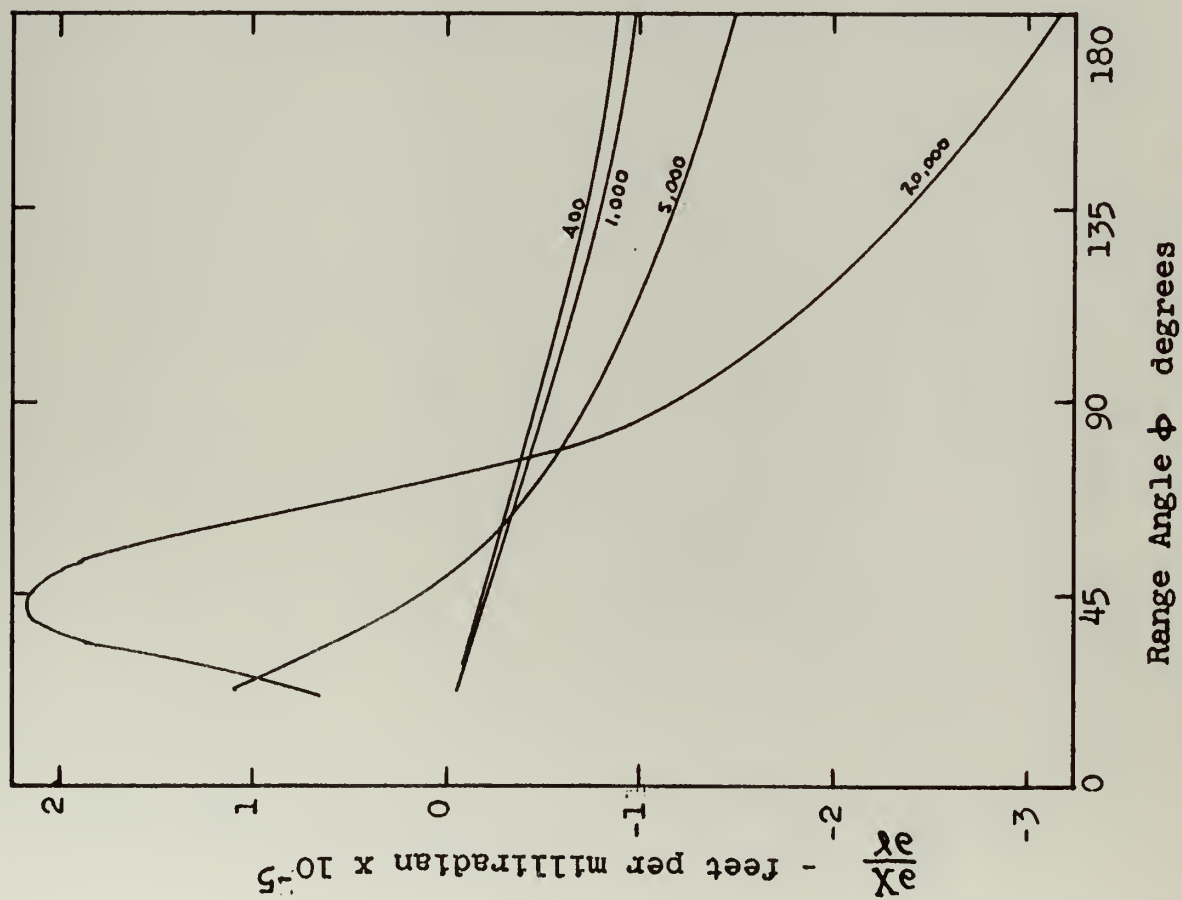


Fig. 12. Effects of In-plane Misalignment of Booster Burnout Velocity on Position at Zero-error Apogee Time.

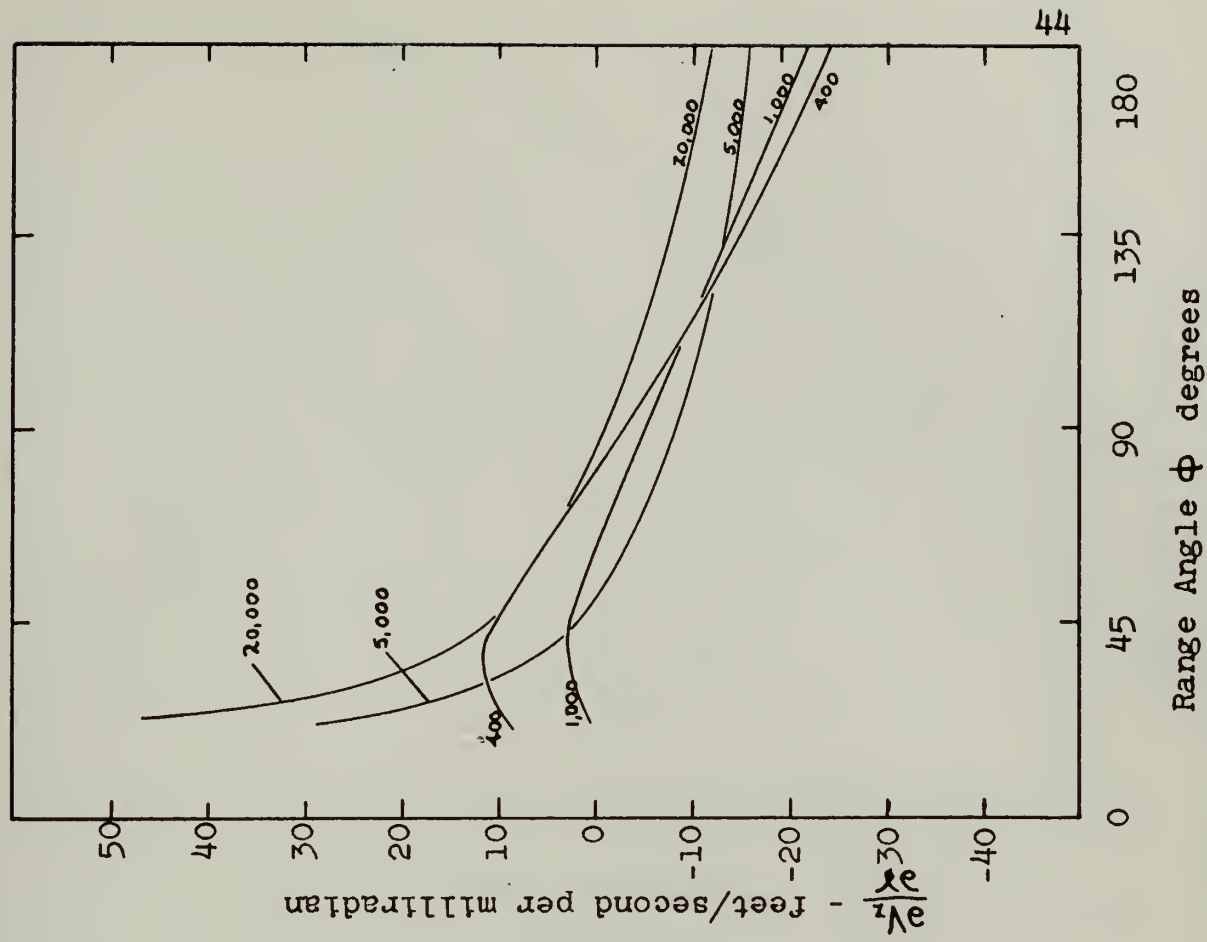
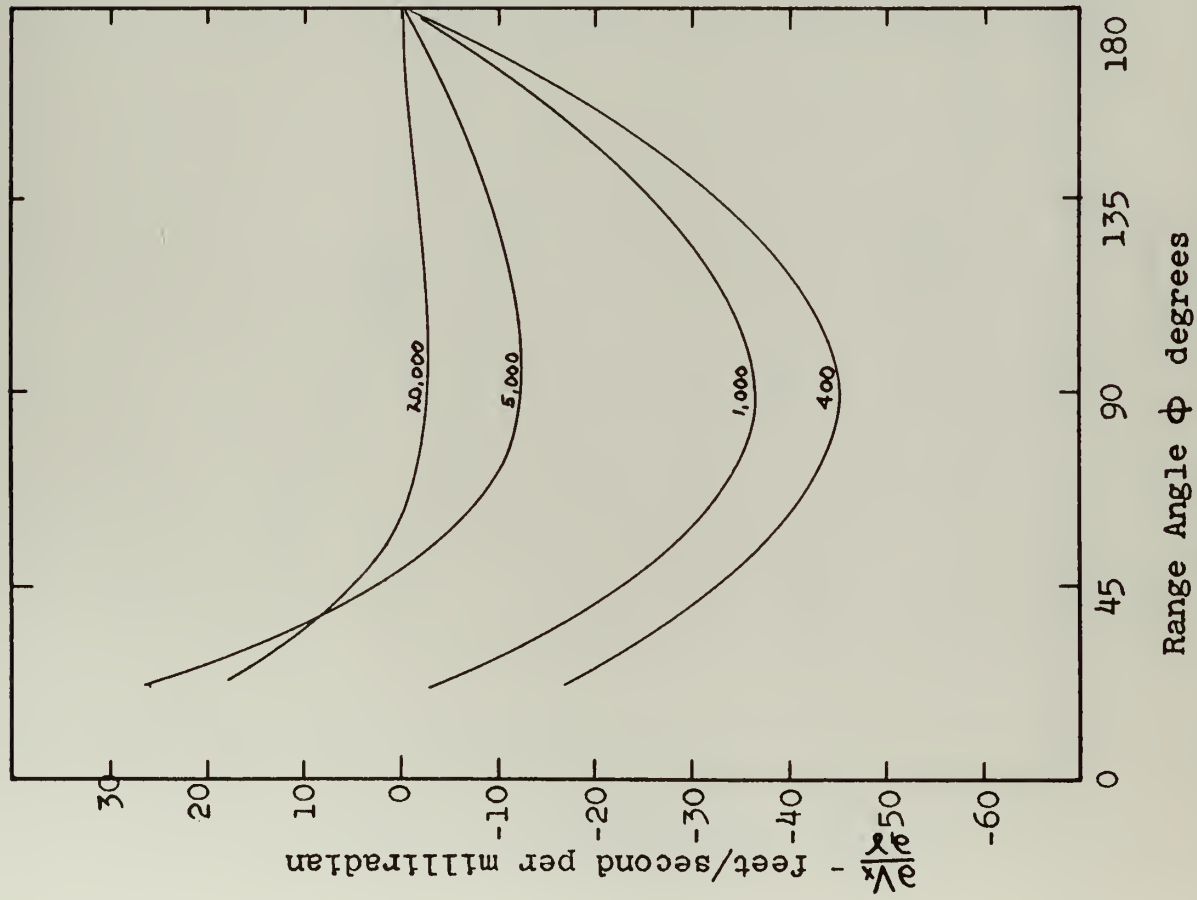


Fig. 13. Effects of In-plane Misalignment of Booster Burnout Velocity on Velocity at Zero-error Apogee Time.

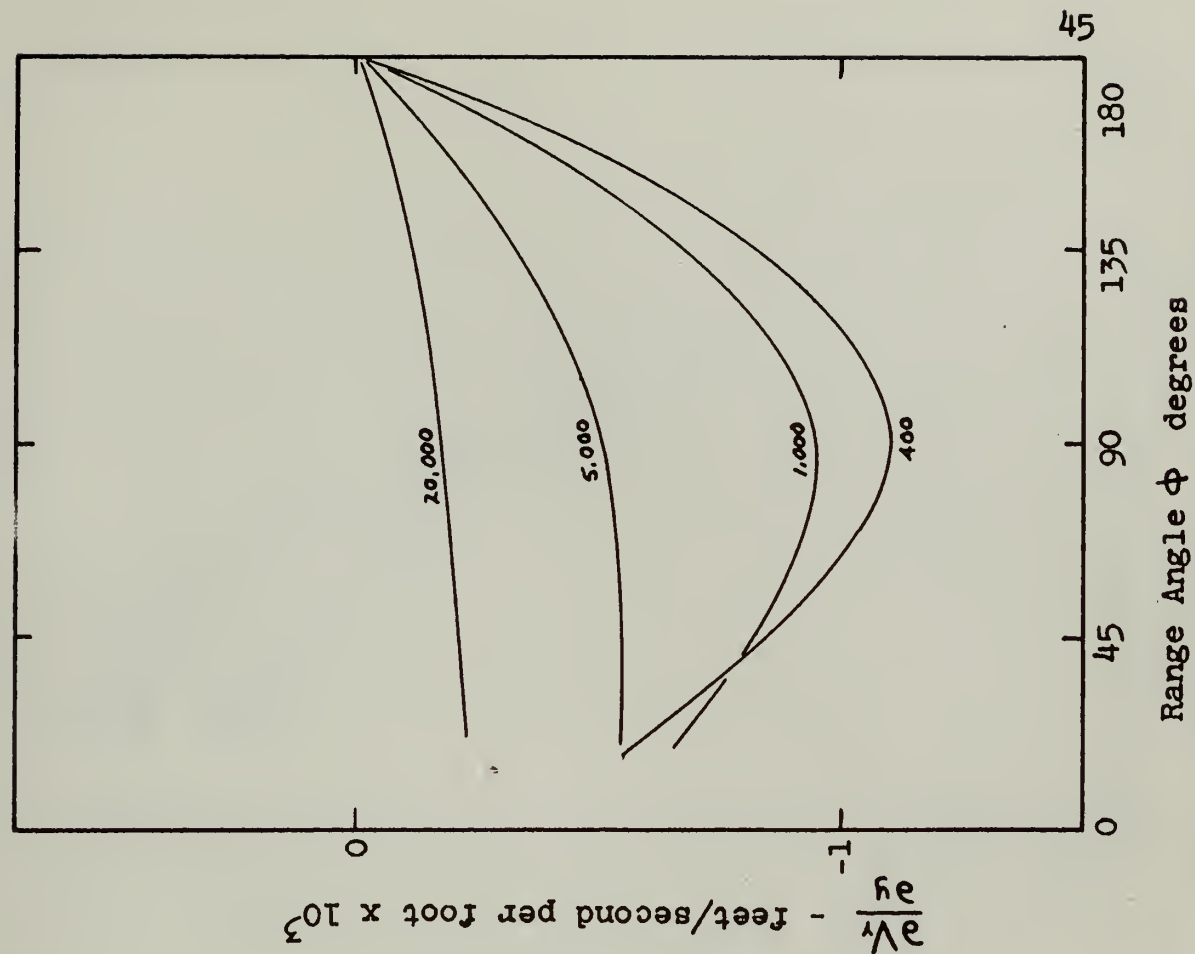
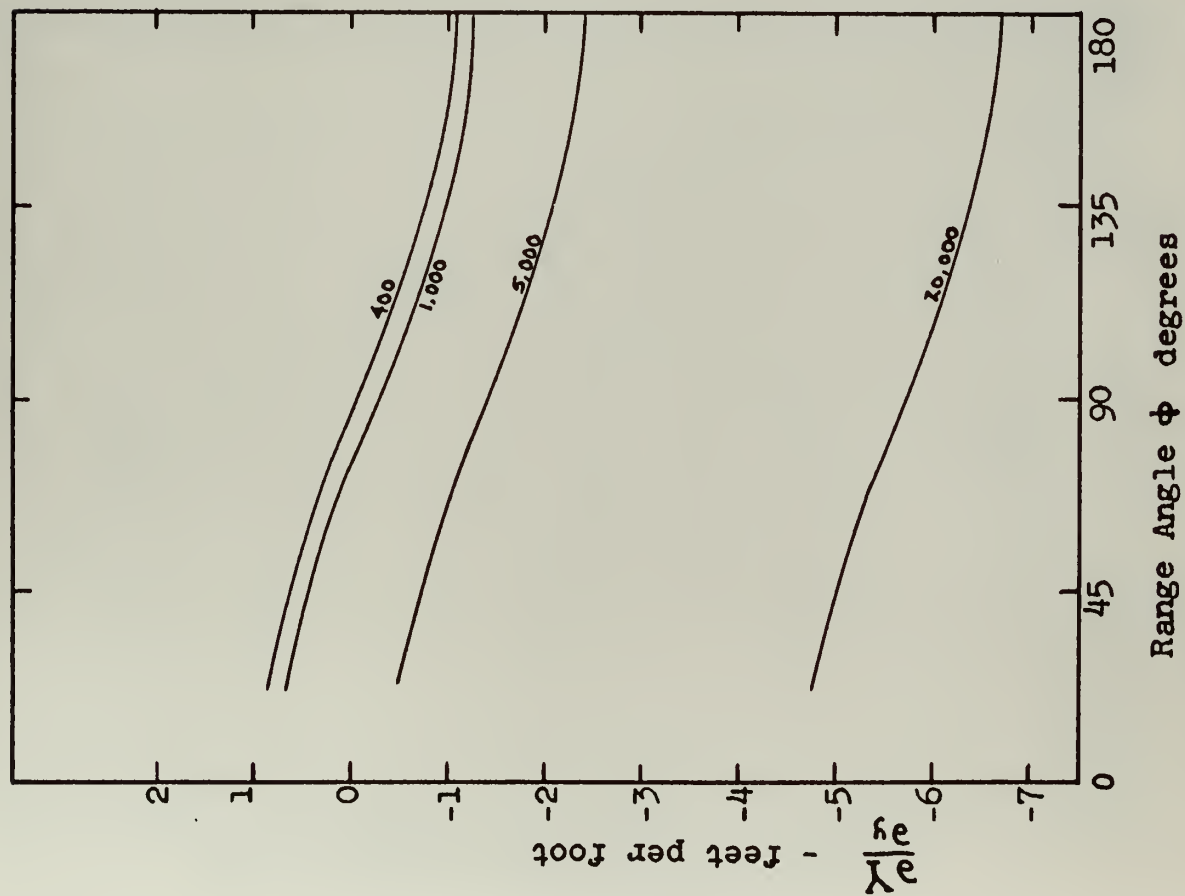


Fig. 14. Effects of Track Error at Booster Burnout on Position and Velocity at Zero-error Apogee Time.

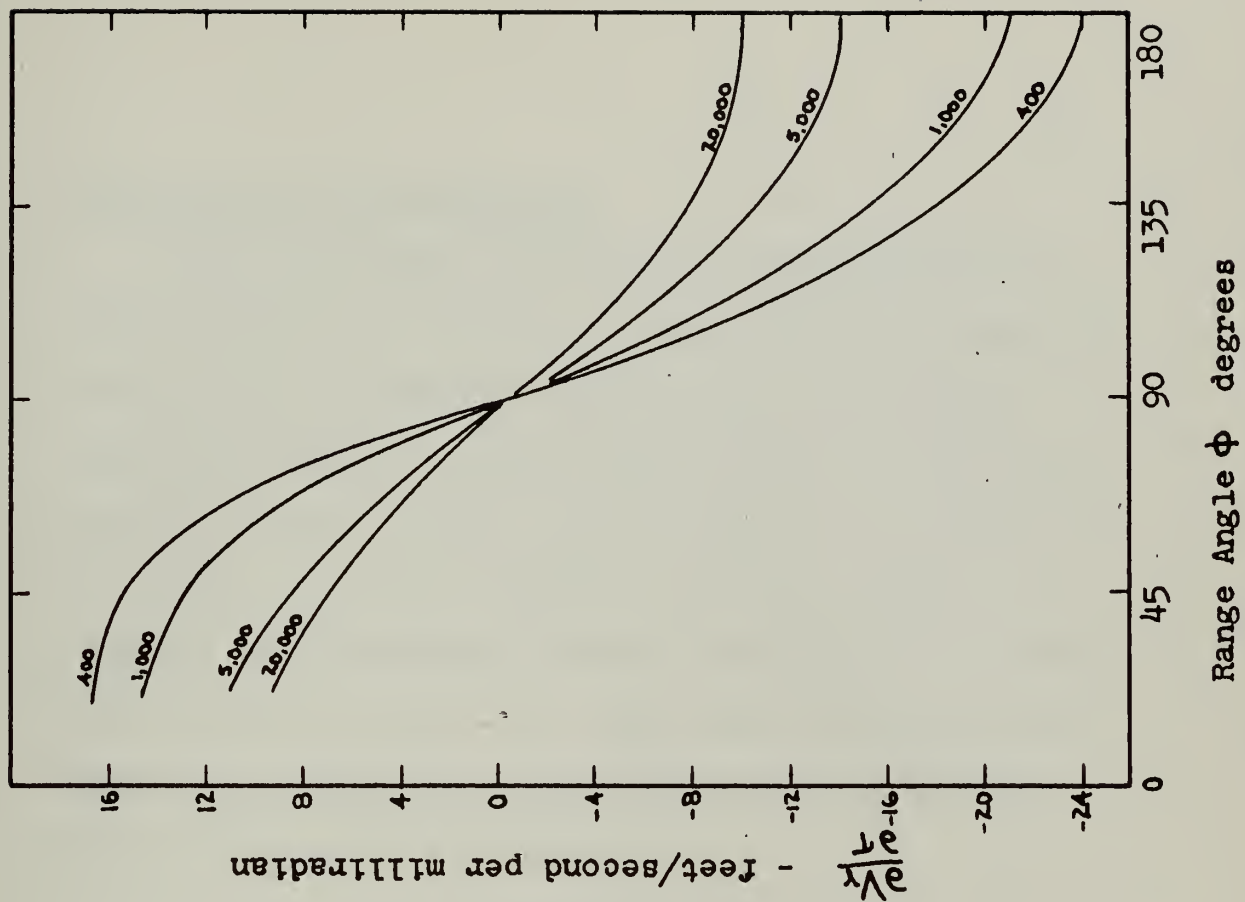
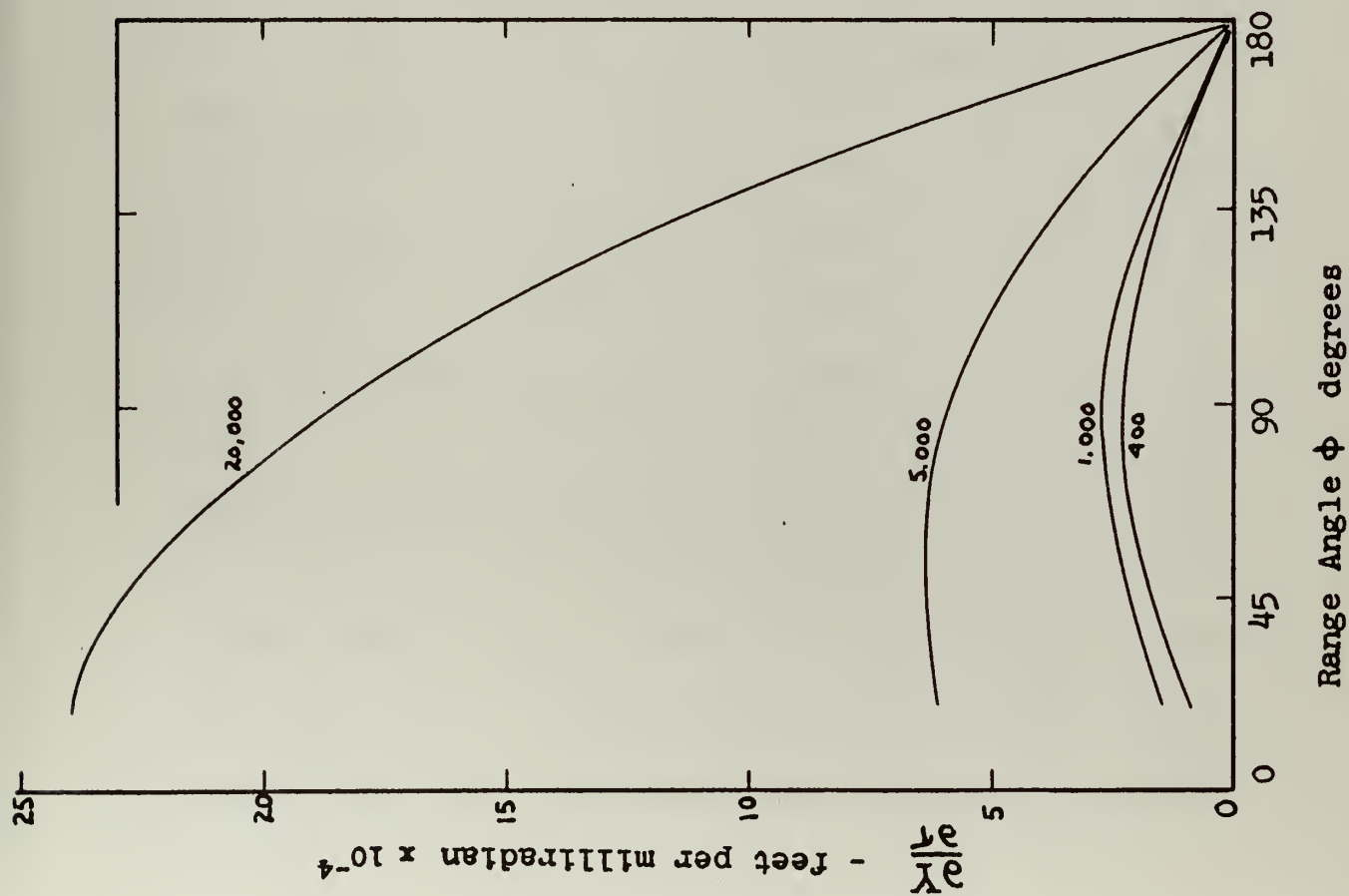


Fig. 15. Effects of Lateral Misalignment of Booster Burnout Velocity on Position and Velocity at Zero-error Apogee Time.

Independence of Derivatives

A final important result is the independence of the in-plane derivatives. To investigate all possible combinations of booster burnout errors in varying magnitude involves endless computation, but the total error is shown in Table I to be made up of the sum of the errors due to the separate effects. Table I is compiled by introducing a random combination of errors into the 1000 n. mile, 90° range angle trajectory and checking the computed output for component addition from the separate derivatives.

TABLE I

Booster Burnout Error	<u>Final Error Components</u>			
	X feet	Z feet	V _x ft/sec	V _z ft/sec
(1) +20 ft/sec	-44,293	58,693	-79.94	40.25
(2) -2 mr	-103,095	40,404	-73.15	-10.03
(3) 1 mile ahead	-7,711	6,048	-10.92	-.75
Sum of (1) + (2)	-147,388	99,097	-153.09	30.22
Computed Results	-147,387	98,929	-153.10	29.89
Sum of (1) + (3)	-52,004	64,741	-90.86	39.50
Computed Results	-51,994	64,753	-90.87	39.48
Sum of (1) + (2) + (3)	-155,099	105,145	-164.01	29.47
Computed Results	-155,080	104,964	-164.04	29.09

Let the assumed booster burnout errors be designated by δx , δz , δV_1 and $\delta \gamma$ for in-plane errors and by δy and $\delta \tau$ for lateral errors. Now if the final errors are designated by ΔX , ΔZ , ΔV_x and ΔV_z for in-plane errors and by ΔY and ΔV_y for lateral errors then a complete final error for a booster burnout error in, say, speed, range and lateral alignment is:

$$\Delta X = \frac{\partial X}{\partial V_1} \delta V_1 + \frac{\partial X}{\partial x} \delta x$$

$$\Delta Z = \frac{\partial Z}{\partial V_1} \delta V_1 + \frac{\partial Z}{\partial x} \delta x$$

$$\Delta V_x = \frac{\partial V_x}{\partial V_1} \delta V_1 + \frac{\partial V_x}{\partial x} \delta x$$

$$\Delta V_z = \frac{\partial V_z}{\partial V_1} \delta V_1 + \frac{\partial V_z}{\partial x} \delta x$$

$$\Delta Y = \frac{\partial Y}{\partial \tau} \delta \tau$$

$$\Delta V_y = \frac{\partial V_y}{\partial \tau} \delta \tau$$

where the partial derivatives are read from the graph for the appropriate altitude and range angle.

Table I demonstrates that this linearization introduces no more than 1% inaccuracy in the final computed values for a practical case. Special notation seems unnecessary since the procedure is so simple. Any combination is readily written if it is remembered that each initial in-plane error causes four components of final error, two in position and two in velocity. The lateral errors, which are independent of the in-plane error, each cause two final errors, one in TRACK position and one in TRACK velocity.

Lateral Position Error

The lateral position error due to a TRACK error at booster burnout rates special mention because it illustrates an effect due entirely to the definition of position error used, that is, the type of platform assumed. The easiest place to view this effect is at a range angle of 90 degrees. It seems at first that for a lateral position error at booster burnout, the line of nodes between the plane of the actual ascent ellipse and the reference plane would lie 90 degrees away, or at apogee for this case. The integrated results shown in Fig. 14 show this is not the case.

Fig. 16 below illustrates how this comes about with the central angle due to a reasonable position error greatly exaggerated.

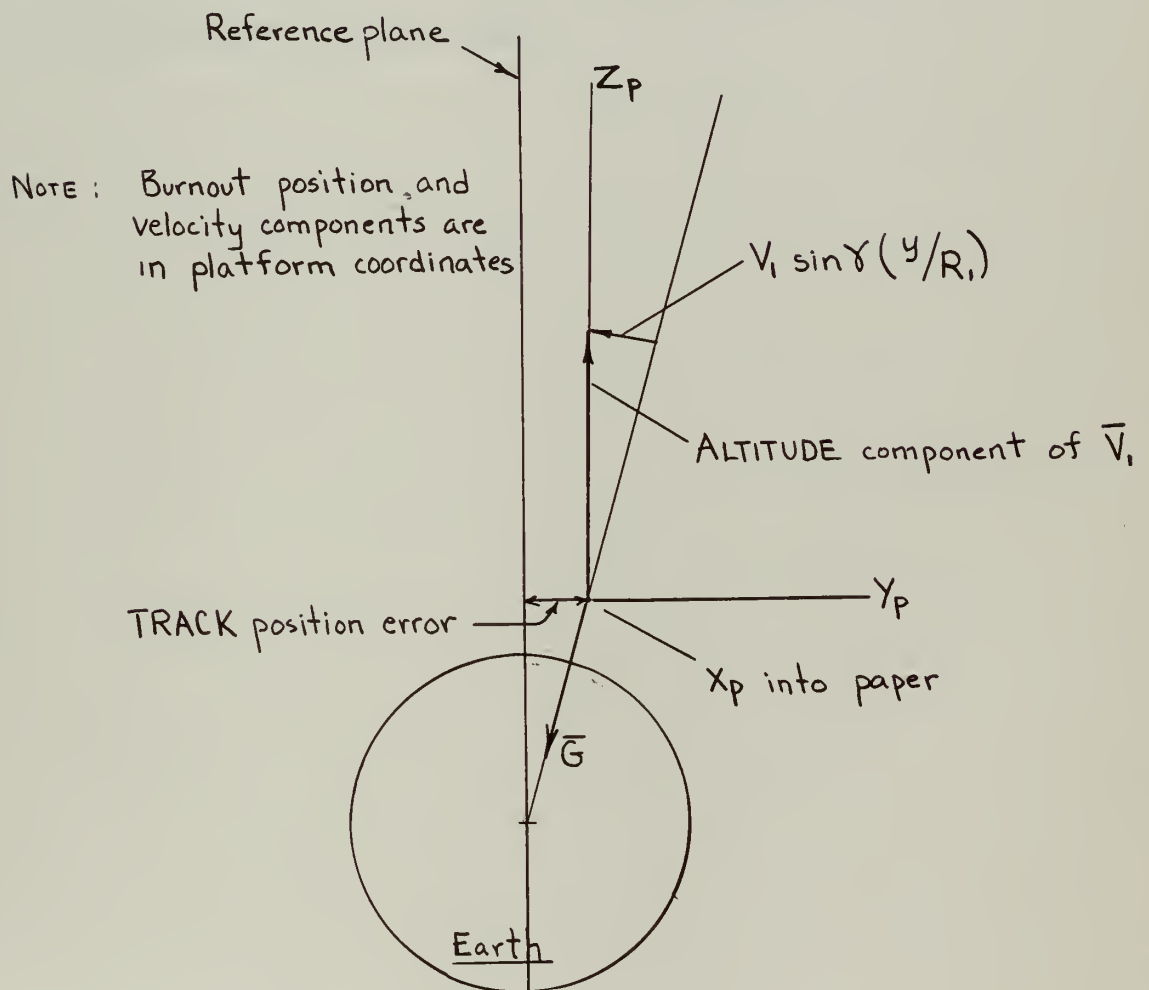


Fig. 16. Effect of Platform Definition on Lateral Position Error.

Note that although no lateral velocity appears in the platform coordinates, with a platform that stays aligned with the reference vertical, a sidewise component of velocity, $V_1 \sin \gamma (y/R_1)$, does show up with respect to the reference orbital plane.

CHAPTER V

CONCLUSIONS

If this thesis has met its objective then the plots showing the various error derivatives should clearly demonstrate the primary conclusions to be drawn concerning the effect of target altitude and range angle on these derivatives.

To review the results in general:

1. For the low altitude rendezvous the summation of booster burnout velocity and injection velocity increment (ΣV) is only slightly greater than the Hohmann value until the range angle is well below 90° . The components of ΣV follow this same pattern.

2. The in-plane error derivatives for a 1000 n. mile target altitude are accurate for differentials of ± 60 ft/sec, ± 2 milliradians and 12,000 feet in any direction.

3. The lateral error derivatives are independent of the in-plane error derivatives.

4. The in-plane partial derivatives are independent of each other and therefore the total error components can be formed by the rule for total differentials.

5. The propulsive requirements for the 20,000 n. mile orbit altitude are shown to be so great that multi-staging will certainly be necessary.

At this altitude the sensitivity to error is so great that special guidance techniques will also be required.

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